Math 33A **Name:** ____________________

ID Number ____________________

Spring 2020
Practice Midterm 2
5/16/2020

This exam contains 9 pages (including this cover page) and 4 problems.

You are required to show your work on each problem on this exam. This holds even if work isn't explicitly asked in the statement of the problem. Please take care to show your steps, providing a reasonable amount of justification for your work that demonstrates your understanding. The following rules apply:

- You don't need to print this out and write directly on the exam, you may submit separate sheets of paper. If you print out and write on the exam, you may use the provided blank pages for your work. In either case, make sure you organize your work, in a reasonably neat and coherent way. Work scattered all over the page without a clear ordering will receive very little credit.
- In problems that have multiple parts, if your answer for a later part depends on a previous part, you can still get partial credit for the later part even if your answer for the previous part is incorrect.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

1. Consider the matrix 4x3 matrix A with linearly independent columns

$$\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

   (a) (3 points) Let $\mathcal{A}$ be the basis of im(A) consisting of the columns of A. Find the orthonormal basis $\mathcal{B}$ obtained by applying Gram-Schmidt to $\mathcal{A}$.

   (b) (4 points) Use the previous part to write down the QR factorization of A, being careful to justify your work.

   (c) (3 points) Explain why we know that the matrix R from any QR factorization must be invertible.

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2. (a) (6 points) Use the method of least squares to find the best fit line to the points

$$(1, 2), (2, 5), (4, 7), (5, 4).$$

   (b) (4 points) Explain why we know that this best fit line minimizes the sum of the squares of the residuals.

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3. (a) (5 points) Recall how transposes of matrices behave with dot products, namely, if A is an nxm matrix and $\vec{v}$ is an m-vector and $\vec{w}$ is an n-vector then

$$(A\vec{v}) \cdot \vec{w} = \vec{v} \cdot (A^T \vec{w}).$$

Use this to show that orthogonal matrices preserve angles between vectors; that is if A is orthogonal and the angle between $\vec{v}$ and $\vec{w}$ is θ, then the angle between $A\vec{v}$ and $A\vec{w}$ is also θ. (Hint: first show that they preserve dot products).

(b) (5 points) Use determinants to answer the following question. Let Q denote the cube in $\mathbb{R}^3$ of sidelength 1 centered at the point $(2, 4, 5)$. If T is the linear transformation with matrix A given below, what does T do to the volume of Q, that is what is the volume of the parallelepiped $T(Q)$?

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{pmatrix}$$

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4. Let A be the matrix

$$\begin{pmatrix} 3 & 0 & 0 \\ -3 & 4 & 9 \\ 0 & 0 & 3 \end{pmatrix}$$

(a) (3 points) Find the eigenvalues of A.

(b) (3 points) Find the corresponding eigenspaces of A. List the algebraic and geometric multiplicities of each eigenvalue.

(c) (4 points) Is A diagonalizable or not? Explain how you know. If so, diagonalize A by finding S, S^{-1}, B so that $A = SBS^{-1}$ and B is diagonal.

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