Dr. Coroniti

...are 100 points on the exam, and you have 50 minutes. To receive full credit, ...reasoning. No credit will be given for answers that simply ... not need calculators, so

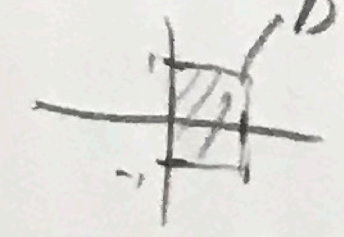
**Math 32B-1 Yeliussizov. Final exam**

Exam time: 8:00 ...

**Problem 1.** (10 points) Evaluate the double integral over the given rectangular domain in the $xy$-plane

$$\iint_R (y^3 + 2 + 3y\sqrt{1+xy})\, dA, \qquad R = [0,1] \times [-1,1].$$

$D = R$



area $= 1 \times 2 = 2$

$$\iint_R y^3\, dA + \iint_R 2\, dA + \iint_R 3y\sqrt{1+xy}\, dA$$

$$\int_{x=0}^{1}\int_{y=-1}^{1} y^3\, dy\, dx$$

$$\int_0^1 \left( \frac{y^4}{4} \Big|_{y=-1}^{y=1} \right) dx$$

$$\int_0^1 \frac{1}{4} - \frac{1}{4}\, dx$$

$$\int_0^1 0$$

$$= 0$$

$2(\text{Area }(R))$

$= 2(2)$

$= 4$

$$\int_{x=0}^{1}\int_{y=-1}^{1} 3y\sqrt{1+xy}\, dy\, dx$$

switch order

$$= 3\int_{y=-1}^{1}\int_{x=0}^{1} y\sqrt{1+xy}\, dx\, dy$$

$u = 1+xy$

$du = y\,dx$

$\frac{1}{y} du = dx$

$u(1) = 1+y$

$u(0) = 1$

$$= 3\int_{y=-1}^{1}\left(\int_{x=0}^{1} u^{\frac{1}{2}}\, du\right) dy$$

$$3\int_{y=-1}^{1}\int_{u=1}^{1+y} u^{\frac{1}{2}}\, du$$

$$= 3\int_{y=-1}^{1}\left( \frac{2}{3} u^{\frac{3}{2}} \Big|_{x=0}^{x=1} \right) dy$$

$$3\int_{-1}^{1}\left( \frac{2}{3} u^{\frac{3}{2}} \Big|_{u=1}^{1+y} \right) dy$$

$$= 3\int_{y=-1}^{1}\left( \frac{2}{3}(1+xy)^{\frac{3}{2}} \Big|_{x=0}^{x=1} \right) dy$$

$$3\int_{-1}^{1} \frac{2}{3} (1+y)^{\frac{3}{2}} - 1\, dy$$

$$= 2\int_{y=-1}^{1} \left( (1+y)^{\frac{3}{2}} - 1\, dy \right)$$

$u = 1+y$ $\quad u(1) = 2$

$du = dy$ $\quad u(-1) = 0$

$$2\int_{-1}^{1} (1+y)^{\frac{3}{2}}\, dy - 2\int_{-1}^{1} dy$$

$$2\int_{u=0}^{2} u^{\frac{3}{2}}\, du - 4$$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$$= 2 \cdot \frac{2}{5} u^{\frac{5}{2}} \Big|_{u=0}^{2} - 4$$

$$= \frac{4}{5}(2)^{\frac{5}{2}} - 4$$

$$4 + \frac{4}{5}(2)^{\frac{5}{2}} - \cancel{4} = \frac{4}{5}(2)^{\frac{5}{2}}$$

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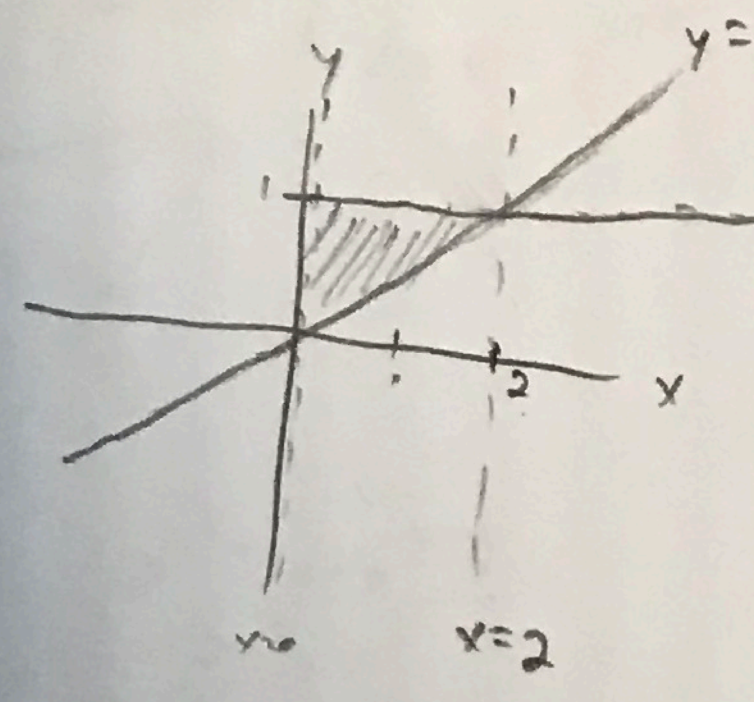
Math 32B-1 Yeliussizov. Final exam

...m time: 8:00 ...

Midterm 2

**Problem 2.** (10 points) Evaluate the integral $\int_0^2 \int_{x/2}^1 e^{y^2}\, dy\, dx$ by changing the order of integration.




$x =$ $y =$

vert simple

$0 \le x \le 2$ $\quad \frac{x}{2} \le y \le 1$

$y = \frac{1}{2}x$

$y = \frac{x}{2}$ $\quad x = 2y$

hor simple: $\quad 0 \le y \le 1$

$0 \le x \le 2y$

$$\int_{y=0}^{1} \int_{x=0}^{2y} e^{y^2}\, dx\, dy$$

$$\int_{y=0}^{1} \left( x e^{y^2} \Big|_{x=0}^{2y} \right) dy = \int_{y=0}^{1} 2y e^{y^2}\, dy$$

$u(1) = 1$
$u(0) = 0$
$u = y^2$
$du = 2y\, dy$

$$\int_{u=0}^{1} e^u\, du = e^u \Big|_{u=0}^{u=1} = e - 1$$

**Midterm 1**

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**32B-1 Yeliussizov. Final exam**

...me: 8:00 ...

**Problem 3.** (10 points) Let $W$ be the region enclosed by $y = x^2$, $y = 1 - x^2$, $z = 1$, and $z + y = 0$. Evaluate $\iiint_W \frac{2}{y+1}\, dV$.

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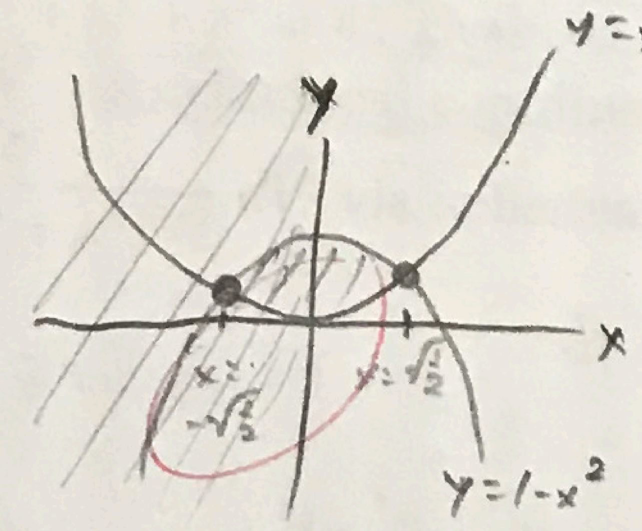


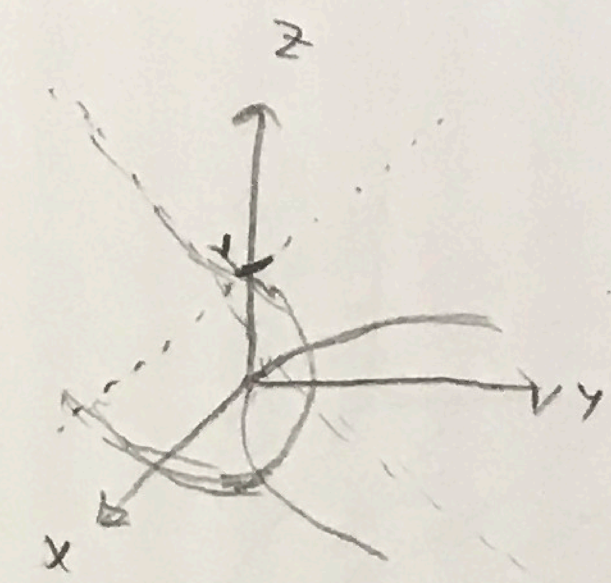


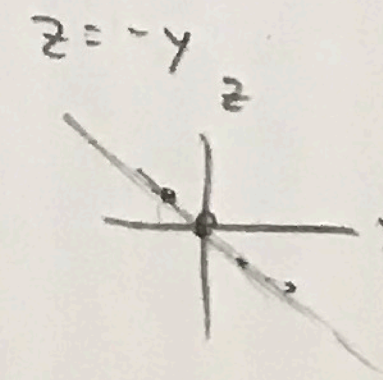


$x^2 = 1 - x^2$

$2x^2 = 1$

$x^2 = \frac{1}{2}$

$x = \sqrt{\frac{1}{2}}$

$$\int_{x=0}^{\sqrt{\frac{1}{2}}} \int_{y=x^2}^{1-x^2} \int_{z=-y}^{z=1} \frac{2}{y+1}\, dz\, dy\, dx \qquad 7 \qquad \frac{2}{y+1} z \Big|_{z=-y}^{z=1}$$

$$\int_0^{\sqrt{\frac{1}{2}}} \int_{x^2}^{1-x^2} \frac{2}{y+1} + \frac{2y}{y+1}\, dy\, dz$$

$$\int_0^{\sqrt{\frac{1}{2}}} \int_{x^2}^{1-x^2} \frac{2(1+y)}{(1+y)}\, dy\, dz = \int_0^{\sqrt{\frac{1}{2}}} \int_{x^2}^{1-x^2} 2\, dy\, dz = 2\int_0^{\sqrt{\frac{1}{2}}} (1-x^2) - x^2$$

$$2\int_0^{\sqrt{\frac{1}{2}}} 1 - 2x^2\, dx = 2\left(x - \frac{2}{3}x^3\right)\Big|_{x=0}^{\sqrt{\frac{1}{2}}}$$

$$= 2\left(\sqrt{\tfrac{1}{2}} - \frac{2}{3}\left(\tfrac{1}{2}\right)^{\frac{3}{2}}\right)$$

PHYSICS 1A

Midterm 1

Dr. Coroniti

oints on the exam, and you have 50 minutes. To receive full credit,
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**Problem 4.** (15 points) Let $W$ be the region in the first octant $x, y, z \geq 0$ inside the cylinder $x^2+y^2=1$ and bounded by the sphere $x^2+y^2+z^2=4$. Evaluate

(a) (7 points) the volume of $W$ via cylindrical coordinates

(b) (8 points) $\iiint_W \frac{z}{(x^2+y^2+z^2)^{3/2}}\, dV$ via spherical coordinates

$\rho = 2$

$r^2 = 1$

$r = 1$

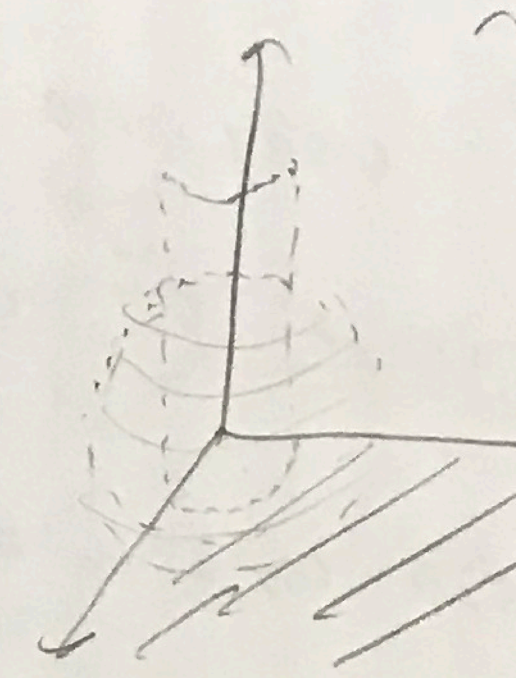
$z = \sqrt{4-r^2}$ $\quad dV = r\,dz\,dr\,d\theta$

$x^2+y^2+z^2=4$

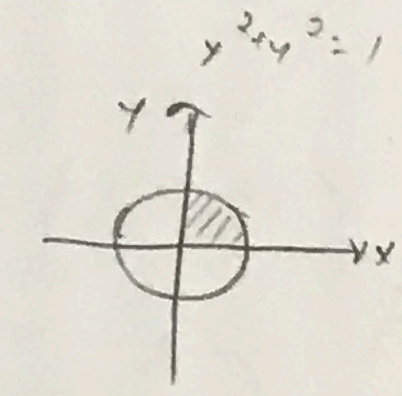
$\sqrt{z^2} = \sqrt{4-r^2}$

$z = \sqrt{4-r^2}$

(a) $\int_{\theta=0}^{\frac{\pi}{2}} \int_{r=0}^{1} \int_{z=0}^{\sqrt{4-r^2}} r\, dz\, dr\, d\theta$



$x^2+y^2=1$

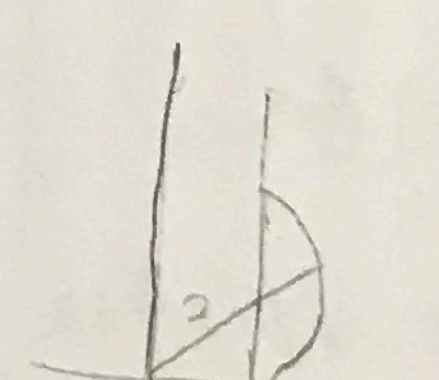


$\int_0^{\frac{\pi}{2}} \int_0^1 \left( rz \Big|_{z=0}^{\sqrt{4-r^2}} \right) dr\, d\theta$

$\int_0^{\frac{\pi}{2}} \int_0^1 r\sqrt{4-r^2}\, dr\, d\theta$ $\quad u = 4-r^2$ $\quad u(1) = 3$

$du = -2r\,dr$ $\quad u(0) = 4$

$-\frac{1}{2} du = r\,dr$



$\int_0^{\pi/2} \left(-\frac{1}{2}\right) \int_{u=4}^{3} u^{\frac{1}{2}} du \Big) d\theta$ $\quad -\frac{1}{3}\pi\left(3^{\frac{3}{2}} - 4^{\frac{3}{2}}\right) = -\frac{\pi}{3}\left(3\sqrt{3} - 8\right)$

$\int_0^{\pi/2} \left(-\frac{1}{2}\right)\left(\frac{2}{3}\right) u^{\frac{3}{2}} \Big|_{u=4}^{u=3} d\theta = \int_0^{\pi} -\frac{1}{3}\left(3^{\frac{3}{2}} - 4^{\frac{3}{2}}\right) d\theta$

$= -\frac{1}{3}\left(3\sqrt{3}-8\right)\theta \Big|_{\theta=0}^{\pi} = \left(\frac{8}{3} - \sqrt{3}\right)\pi$

(b) $x^2+y^2+z^2=4$ $\quad \rho^2 = 4$

$\rho = 2$ $\quad dV = \rho^2 \sin\phi$ $\quad r = \rho\sin\phi$

$x^2+y^2=1$ $\quad \frac{z}{(\rho^2)^{\frac{3}{2}}} = \frac{\rho\cos\phi}{\rho^3}$

$0 \leq \theta \leq \frac{\pi}{2}$ $\quad r^2 = 1$

$0 \leq \phi \leq \frac{\pi}{2}$ $\quad \rho^2 \sin^2\phi = 1$ $\quad \rho = \sqrt{\frac{1}{\sin^2\phi}}$ $\quad = \frac{\cos\phi}{\rho^2}$

$\rho^2 = \frac{1}{\sin^2\phi}$

$\int_{\theta=0}^{\frac{\pi}{2}} \int_{\phi=0}^{\frac{\pi}{2}} \int_{\rho=\sqrt{\frac{1}{\sin^2\phi}}}^{2} \frac{\cos\phi}{\rho^2}\, \rho^2 \sin\phi\, d\rho\, d\phi\, d\theta = \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \int_{\sqrt{\frac{1}{\sin^2\phi}}}^{2} \cos\phi\sin\phi\, d\rho\, d\phi\, d\theta$

$= \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \underbrace{2\cos\phi\sin\phi}_{\sin 2\phi} - \sqrt{\frac{1}{\sin^2\phi}} \cos\phi\sin\phi\, d\phi\, d\theta \Rightarrow \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sin 2\phi\, d\phi\, d\theta - \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} \sqrt{\frac{1}{\sin^2\phi}} \cos\phi\sin\phi\, d\phi\, d\theta$

on back

$e^{i\theta} = \cos\theta + i\sin\theta$

$\cos 2\theta + i\sin 2\theta = e^{i2\theta} = \cos^2\theta - \sin^2\theta + 2i\sin\theta\cos\theta$

$-4$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\sqrt{\frac{1}{\sin^2\theta}} = \frac{1}{\sin\theta}$

$$\int_{\theta=0}^{\frac{\pi}{2}}\int_{\phi=0}^{\frac{\pi}{2}} \sin 2\phi \, d\phi\, d\theta - \int_{\theta=0}^{\frac{\pi}{2}}\int_{\phi=0}^{\frac{\pi}{2}} \frac{1}{\sin\phi}\cos\phi\sin\phi\, d\phi\, d\theta$$

$$\int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}} \sin 2\phi\, d\phi\, d\theta - \int_0^{\frac{\pi}{2}}\int_0^{\frac{\pi}{2}}\cos\phi\, d\phi\, d\theta$$

$2$

$u = 2\phi$ $\quad u(\frac{\pi}{2}) = \pi$

$du = 2d\phi$ $\quad u(0) = 0$

$\frac{1}{2}du = d\phi$

$$\int_{\theta=0}^{\frac{\pi}{2}}\left(\int_{u=0}^{\pi} \sin(u)\,du\right)d\theta - \int_{\theta=0}^{\frac{\pi}{2}}\left(\sin\phi\Big|_{\phi=0}^{\frac{\pi}{2}}\right)d\theta$$

$$\frac{\pi}{2}(-\cos u)\Big|_{u=0}^{\pi} - \frac{\pi}{2}\left(\sin\frac{\pi}{2} - \sin 0\right)$$

$$\frac{\pi}{2}(-(-1)+1) - \frac{\pi}{2}(1)$$

$$\frac{\pi}{2}(2) - \frac{\pi}{2} = \frac{\pi}{2}$$

PHYSICS 1A

Winter, 2017

Midterm 1

Dr. Coroniti

...are 100 points on the exam, and you have 50 minutes. To receive full credit,
...ning. No credit will be given for ...

**Problem 5.** (15 points) Let $W$ be the tetrahedron with vertices $(1,0,0)$, $(1,1,0)$, $(0,1,0)$, and $(0,1,1)$.
(a) (8 points) Find the volume of $W$.
(b) (7 points) Find the $z$-coordinate of the centroid of $W$ (i.e., the center of mass assuming the mass density $\delta(x,y,z) = 1$).

uniform density

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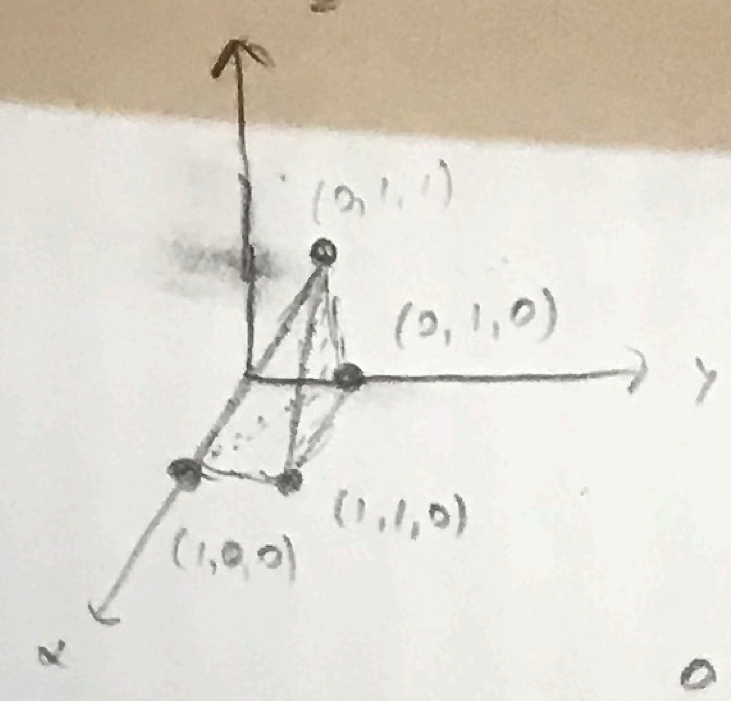

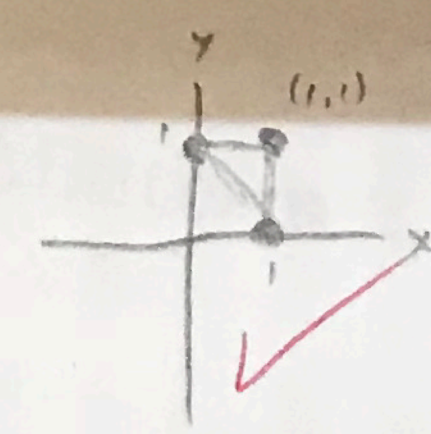

$0 \le x \le 1$

$1 \le y \le 1-x$

$ax + by + cz = 1$

$0 \le z \le$

$0 \le y \le 1$

$1 \le y \le 1-x$

$\langle 1,-1,-1\rangle \times \langle 1,0,-1\rangle$

$\int_{x=0}^{1}\int_{y=1-x}^{1}\int_{z=0}^{?} dz\,dy\,dx$

$\begin{vmatrix} i & j & k \\ 1 & -1 & -1 \\ 1 & 0 & -1 \end{vmatrix} = i(1) - j(0) + k(1)$

$(x-r)\cdot n = 0$

$(x-1, y+z)(-1,-2,1) = 0$

$-x+1-2y+z=0$

$x-1+z=0$

$x+z=1$

$z = 1-x$ ← equation of plane

$(1)i - j(0) + k(1)$

$\langle 1,0,1\rangle$

$\int_{x=0}^{1}\int_{y=1-x}^{1}\int_{z=0}^{1-x} dz\,dy\,dx$

$\int_0^1\int_{1-x}^{1} 1-x \; dy\,dx = \int_0^1 \left((1-x)y\Big|_{y=1-x}^{1}\right)dx = \int_0^1 (1-x) - (1-x)^2\,dx$

$x^2-2x+1$

$\int_0^1 1-x-x^2+2x-1\,dx = \int_0^1 -x^2+x\,dx$

$y^2-2x+1-1+x$

$y^2-x$

$-\frac{x^3}{3}+\frac{x^2}{2}\Big|_{x=0}^{1} = -\frac{1}{3}+\frac{1}{2}$

$-\frac{2}{6}+\frac{3}{6} = \frac{1}{6}$

4

(b) $\dfrac{\iiint_W z(1)\,dW}{\iiint_W \delta\,dW}$

$\int_0^1\int_{1-x}^{1}\int_0^{1-x} z\,dz\,dy\,dx = \int_0^1\int_{1-x}^{1} \frac{z^2}{2}\Big|_{z=0}^{1-x} dy\,dx = \frac{1}{2}\int_0^1\int_{1-x}^{1} (1-x)^2\,dy\,dx$

$= \frac{1}{2}\int_0^1 (1-x)^2 y\Big|_{y=1-x}^{1} = \frac{1}{2}\int_0^1 (1-x)^2 - (1-x)^3\,dx$

$(1-x)^2 = (x^2-2x+1)(1-x)$

$x^2-2x+1-x^3+2x^2-x =$

$-x^3+3x^2-3x+1$

$\frac{1}{2}\int_0^1 -x^3+3x^2-3x+1$

$\frac{1}{2}\left(-\frac{1}{4}x^4 - x^3 - \frac{3}{2}x^2 + x\Big|_0^1\right)$

$\frac{1}{2}\left(-\frac{1}{4}-1-\frac{3}{2}+1\right)$

$\frac{1}{2}\left(\frac{7}{4}\right) = \frac{7}{8}$

$\rho$ found in part A

$\dfrac{\frac{7}{8}}{\frac{1}{6}} =$

$\frac{42}{8}$

F Thu