MECH&AE

PHYSICS 1A

Midterm 1

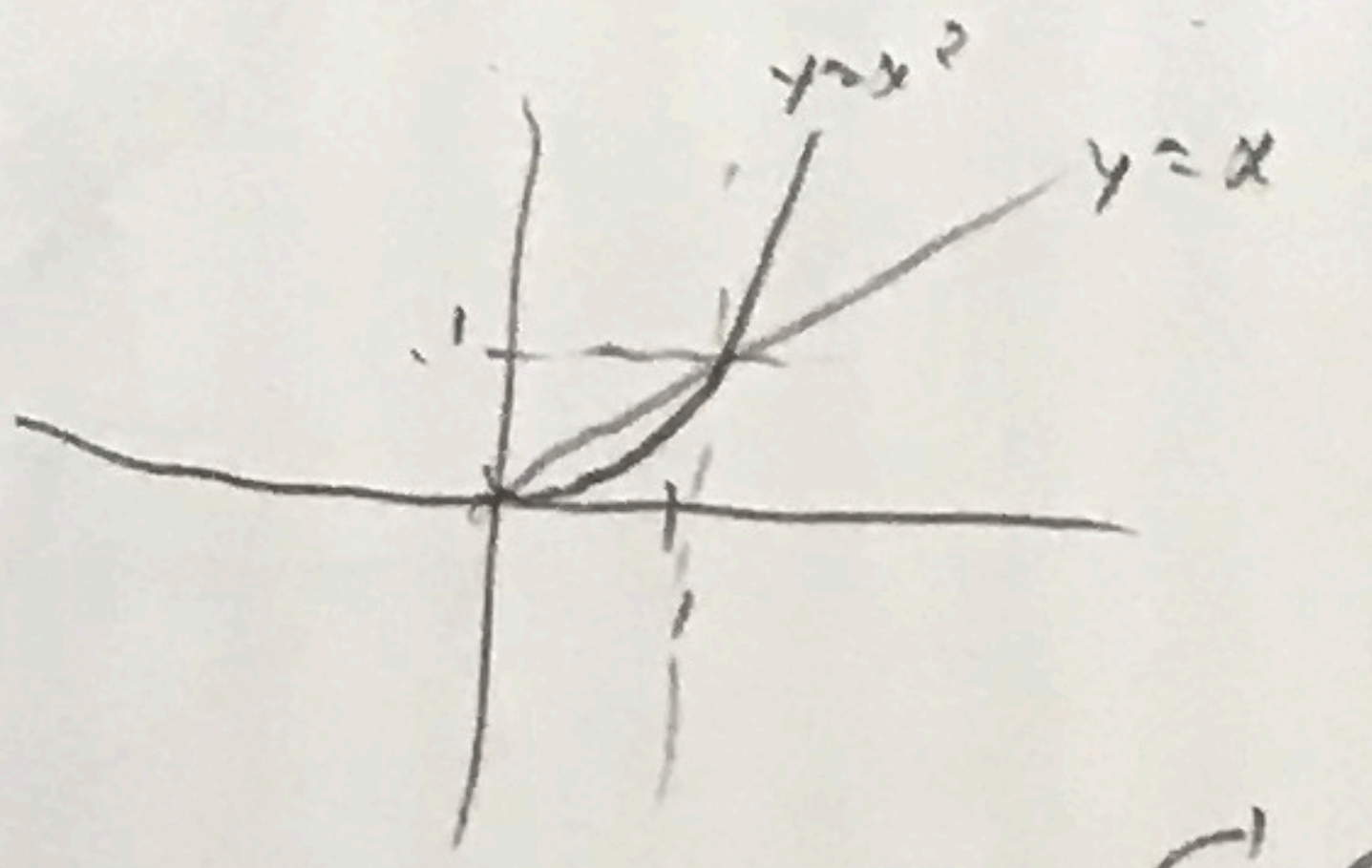
Dr. Coroniti

0 points on the exam, and you have 50 minutes. To receive full credit,
rk and reasoning. No credit will be given for answers that simply
exam is closed notes and closed book. You do not need calculators, so
em. and all cell phones, away. If you need more space, use the backside of

Problem 1. (10 points) Let D be the domain bounded by $x = 0$, $x = 1$, $y = x$, and $y = x^2$.
(a) (5 points) Find the area of D.
(b) (5 points) Find the x-coordinate of the centroid of D.

$\iint x\delta\, dx\, dy$

(a)



$0 \le x \le 1$

$y^2 \le y \le x$

$$\int_{x=0}^{1}\int_{y=x^2}^{x} dy\, dx = \int_{x=0}^{1}\left(y\Big|_{x^2}^{x}\right)dx$$

$$= \int_0^1 (x - x^2)\, dx$$

$$\text{Area} = \frac{x^2}{2} - \frac{y^3}{3}\Big|_0^1 = \frac{1}{2} - \frac{1}{3} - 0$$

$$\text{Area} = \frac{3}{6} - \frac{2}{6} = \boxed{\frac{1}{6}}$$

(b)

$$\int_{x=0}^{1}\int_{y=x^2}^{x} x\, dy\, dx \qquad \text{Centroid} \quad \frac{\iint x\, \delta x dy}{\text{Area}}$$

$$\int_{x0}^{1}\left(xy\Big|_{y=x^2}^{x}\right)dx = \int_{x=0}^{1}(x^2 - x^3)\, dx$$

$$\frac{x^3}{3} - \frac{x^4}{4}\Big|_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \boxed{\frac{1}{12}}$$

$$x,\ \text{Centroid} = \frac{\frac{1}{12}}{\frac{1}{6}} = \frac{6}{12} = \boxed{\frac{1}{2} = x \text{ coordinate of centroid}}$$

PHYSICS 1A

Midterm 1

017

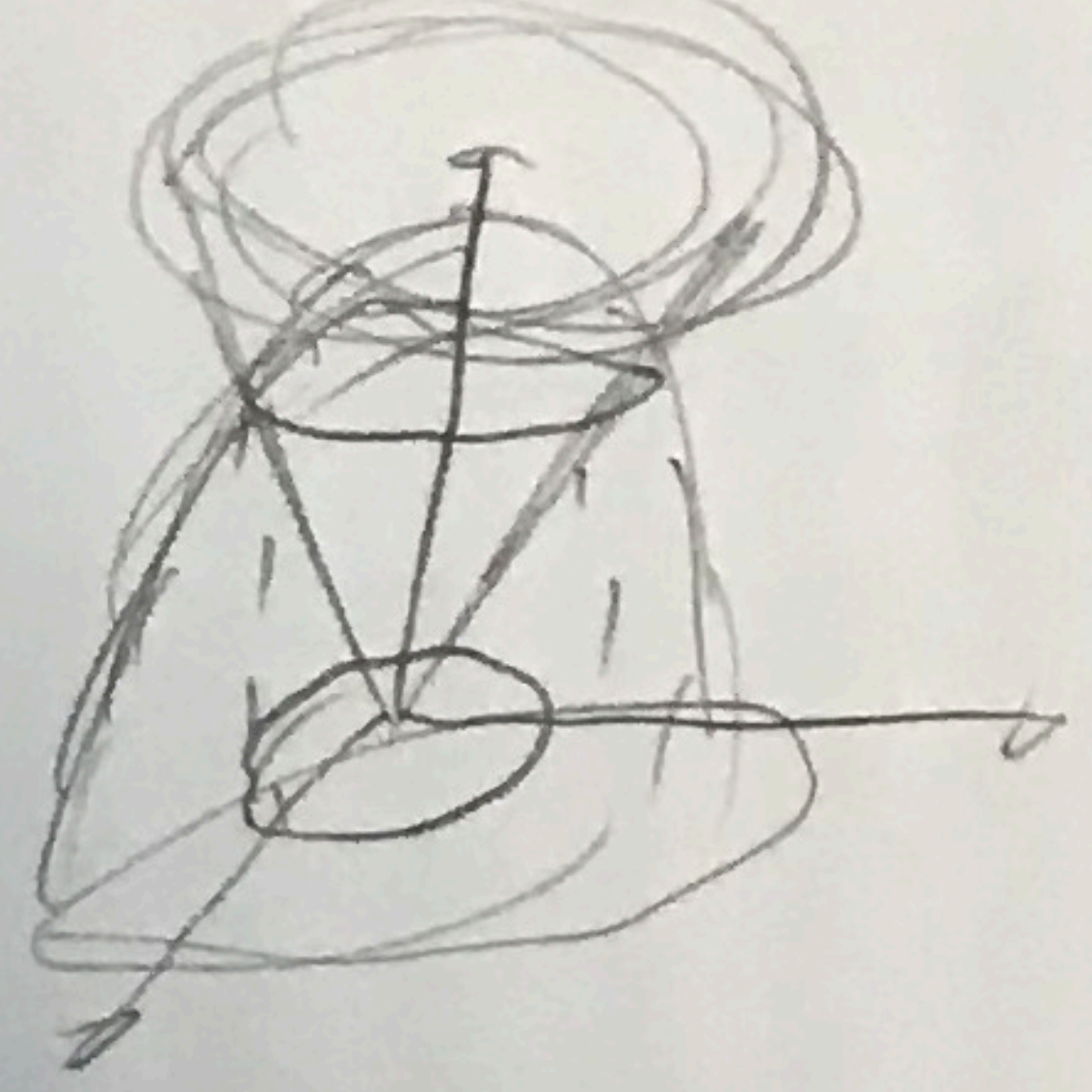
Dr. Coroniti

re are 100 points on the exam, and you have 50 minutes. To receive full credit, you work and reasoning. No credit will be given for answers that simply ". The exam is closed notes and closed book. You do not need calculators, so ll cell phones, away. If you need more space, use the backside of

Problem 2. (10 points) Find the $\boxed{\text{volume}}$ of the solid enclosed by the paraboloid $z = 6 - x^2 - y^2$ and the cone $z = \sqrt{x^2 + y^2}$.

$z = r$

$z = 6 - r^2$



$6 - r^2 = r$

$r^2 + r - 6$

$(r+3)(r-2)$

$\boxed{r = 2}$

-6

$3 \quad -2$

2

$\sqrt{x^2+y^2} \le z \le 6 - x^2 - y^2$

$$\iiint_W dA = \iint_D \left(\int_{z=\sqrt{x^2+y^2}}^{6-x^2-y^2} dz \right) dA$$

$$\iint_D 6 - x^2 - y^2 - \sqrt{x^2+y^2} \, dA$$

$$\int_{\theta=0}^{2\pi}\int_{r=0}^{2} (6 - r^2 - r) \, r \, dr \, d\theta$$

✓

$$\int_0^{2\pi}\int_{r=0}^{2} 6r - r^3 - r^2 \, dr \, d\theta$$

$$2\pi \left(\frac{6r^2}{2} - \frac{r^4}{4} - \frac{r^3}{3} \Big|_0^2 \right)$$

$2 \cdot 2 \cdot 2 \cdot 2$

16

$$2\pi \left(3r^2 - \frac{r^4}{4} - \frac{r^3}{3} \Big|_0^2 \right)$$

$$2\pi \left(3(4) - \frac{2^4}{4} - \frac{2^3}{3} \right)$$

$$2\pi \left(12 - \frac{16}{4} - \frac{8}{3} \right)$$

$$2\pi \left(12 - 4 - \frac{8}{3} \right) = 2\pi \left(8 - \frac{8}{3} \right) = 2\pi \left(\frac{24}{3} - \frac{8}{3} \right)$$

✓

$$2\pi \left(\frac{16}{3} \right) = \boxed{\frac{32\pi}{3}}$$

MECH&AE 10

PHYSICS 1A

Midterm 1

17 Dr. Coroniti

are 100 points on the exam, and you have 50 minutes. To receive full credit, ou work and reasoning. No credit will be given for answers that simply . The exam is closed notes and closed book. You do not need calculators, so all cell phones, away. If you need more space, use the backside of

$\cos 2\theta = \cos^2 - \sin^2$

$\cos 2\theta = 1 - 2\sin^2\theta$

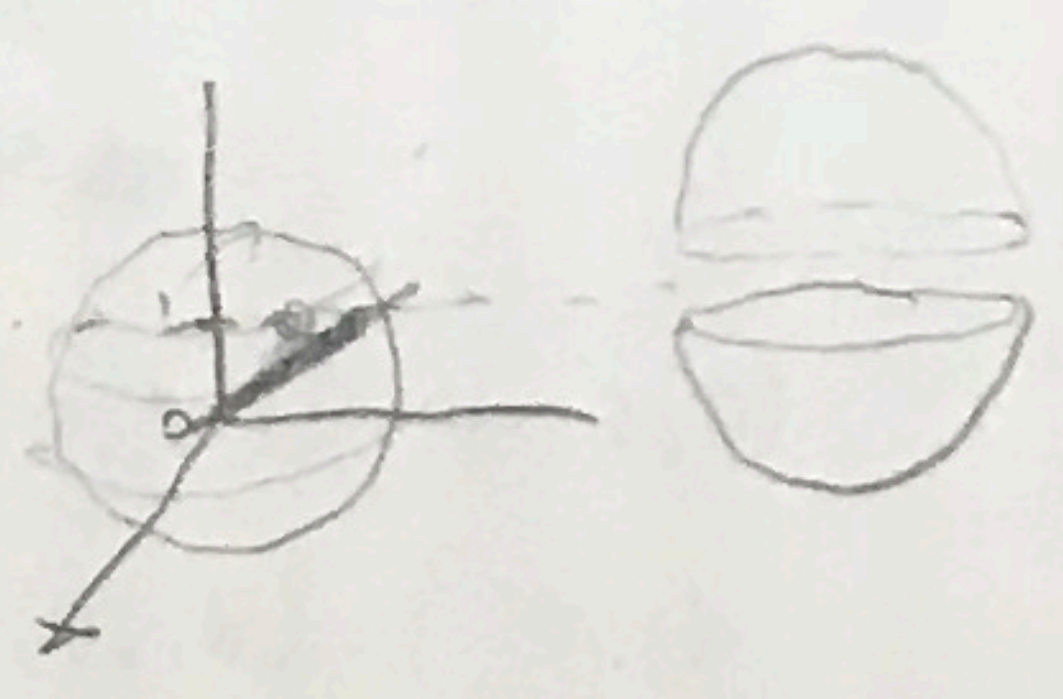
$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

Problem 3. (10 points) Let S be the sphere of radius 2 centered at the origin. Evaluate the surface area of the portion of S between $\boxed{z=0}$ and $z=1$.

$x^2+y^2+z^2=4$

$(x, y, \sqrt{4-x^2-y^2})$

$\iint_S f\,dS$



$G(\theta,\phi) = (2\cos\theta\sin\phi, 2\sin\theta\sin\phi, 2\cos\phi)$

need $N = \|G_\theta \times G_\phi\|\,dA$

$0 \le \theta \le 2\pi$

$\le \phi \le \frac{\pi}{2}$

$G_\theta = (-2\sin\theta\sin\phi, 2\cos\theta\sin\phi, 0)$

$G_\phi = (2\cos\theta\cos\phi, 2\sin\theta\cos\phi, -2\sin\phi)$

$$G_\theta \times G_\phi = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2\sin\theta\sin\phi & 2\cos\theta\sin\phi & 0 \\ 2\cos\theta\cos\phi & 2\sin\theta\cos\phi & -2\sin\phi \end{vmatrix}$$

$G_\theta \times G_\phi = \langle -4\cos\theta\sin^2\phi, -4\sin\theta\sin^2\phi, -4\sin^2\theta\sin\phi\cos\phi - 4\cos^2\theta\sin\phi\cos\phi\rangle$

$G_\theta \times G_\phi = \langle \underset{A}{-4\cos\theta\sin^2\phi}, \underset{B}{-4\sin\theta\sin^2\phi}, \underset{C}{-4\sin\phi\cos\phi}\rangle = e_r$

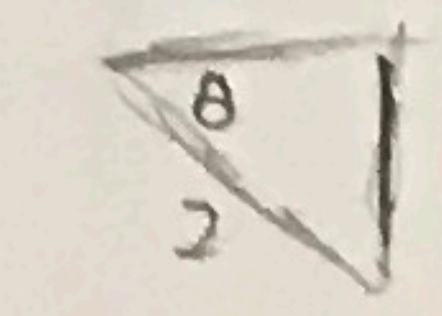
$\|G_\theta \times G_\phi\| = \sqrt{A^2+B^2+C^2} = \boxed{4\sin\phi\, e_r}$

$\sqrt{16\cos^2\theta\sin^4\phi + 16\sin^2\theta\sin^4\phi + 16\sin^2\phi\cos^2\phi}$

$\sqrt{16\sin^4\phi(\cancel{\sin^2+\cos^2}) + 16\sin^2\phi\cos^2\phi}$

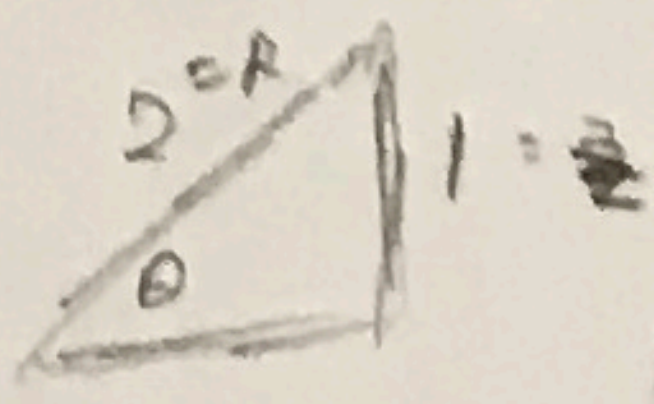
$\sqrt{16\sin^2\phi\,(\sin^2\phi+\cos^2\phi)}$ $=1$

$= 4\sin\phi\, e_r$



$(\cos, \sin)$

$\int_{\theta=0}^{2\pi}\int_{\phi=\frac{\pi}{3}}^{\frac{\pi}{2}} f(G(\theta,\phi)) \cdot \|G_\theta \times G_\phi\|\, d\phi\, d\theta = \int_{\theta=0}^{2\pi}\int_{\phi=\frac{\pi}{3}}^{\frac{\pi}{2}} (2\cos\theta\sin\phi, 2\sin\theta\sin\phi, 2\cos\phi) \cdot 4\sin\phi\,(e_r)\, d\phi\, d\theta$



$(\frac{\sqrt{3}}{3}, \frac{1}{2})$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$

$\frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

$\int_{\theta=0}^{2\pi}\int_{\phi=\frac{\pi}{3}}^{\frac{\pi}{2}} 8\cos\theta\sin^2\phi + 8\sin\theta\sin^2\phi + 8\sin\phi\cos\phi \; d\phi\, d\theta$

$8\sin^2\phi\,(\sin\theta + \cos\theta)$ 1

$$\boxed{2\pi + 16\pi\left(\frac{1}{3}+\frac{\sqrt{3}}{2}\right)}$$

$\sin^2\phi$

$=$

$\int_{\theta=0}^{2\pi}\int_{\phi=\frac{\pi}{3}}^{\frac{\pi}{2}} 8\sin^2\phi + 8\sin\phi\cos\phi\, d\phi\, d\theta$

separate

$8\int_{\theta=0}^{2\pi}\int_{\phi=\frac{\pi}{3}}^{\frac{\pi}{2}} \sin^2\phi\, d\phi\, d\theta + 8\int_{\theta=0}^{2\pi}\int_{\phi=\frac{\pi}{3}}^{\frac{\pi}{2}} \sin\phi\cos\phi\, d\phi\, d\theta$

$\frac{1-\cos 2\phi}{2}$

$8(2\pi)\int_{\phi=\frac{\pi}{3}}^{\frac{\pi}{2}} \sin\phi\cos\phi$

$u = \sin\phi$

$du = \cos\phi$

$u(\frac{\pi}{2}) = 1$

$u(\frac{\pi}{3}) = \frac{\sqrt{3}}{2}$

$= 16\pi\left(\frac{u^2}{2}\Big|_{\frac{\sqrt{3}}{2}}^{1}\right) = 16\pi\left(\frac{1}{2} - \frac{\frac{3}{4}}{2}\right)$

$16\pi\left(\frac{1}{2} - \frac{3}{8}\right)$

$16\pi\left(\frac{1}{8}\right) = 2\pi$

$16(2\pi)\int_{\phi=\frac{\pi}{3}}^{\frac{\pi}{2}} 1-\cos 2\phi\, d\phi = 32\pi\int_{\phi=\frac{\pi}{3}}^{\frac{\pi}{2}} d\phi - 32\pi\int_{\phi=\frac{\pi}{3}}^{\frac{\pi}{2}} \cos 2\phi\, d\phi$

$u = 2\theta$

$du = 2d\theta$

$\frac{1}{2}du = d\theta$

$u(\frac{\pi}{2}) = \pi$

$u(\frac{\pi}{3}) = \frac{2\pi}{3}$

$32\pi\left(\frac{\pi}{6}\right) - \frac{32\pi}{2}\int_{u=\frac{2\pi}{3}}^{\pi} \cos u\, du = \frac{16\pi}{3} - 16\pi\sin u\Big|_{\frac{2\pi}{3}}^{\pi} = \frac{16\pi}{3} - 16\pi\left(-\frac{\sqrt{3}}{3}\right) = 16\pi\left(\frac{1}{3}+\frac{\sqrt{3}}{3}\right)$

$= \frac{16\pi}{3}$

PHYSICS 1A

Midterm 1

2017

Dr. Coroniti

e are 100 points on the exam, and you have 50 minutes. To receive full credit,
you work and reasoning. No credit will be given for answers that simply
. The exam is closed notes and closed book. You do not need calculators, so
and all cell phones, away. If you need more space, use the backside of

Problem 4. (10 points) Let C be the path going from $(1,0)$ to $(0,1)$ along the quarter unit circle in the first quadrant and then continuing from $(0,1)$ to $(-2,0)$ along straight line segment.

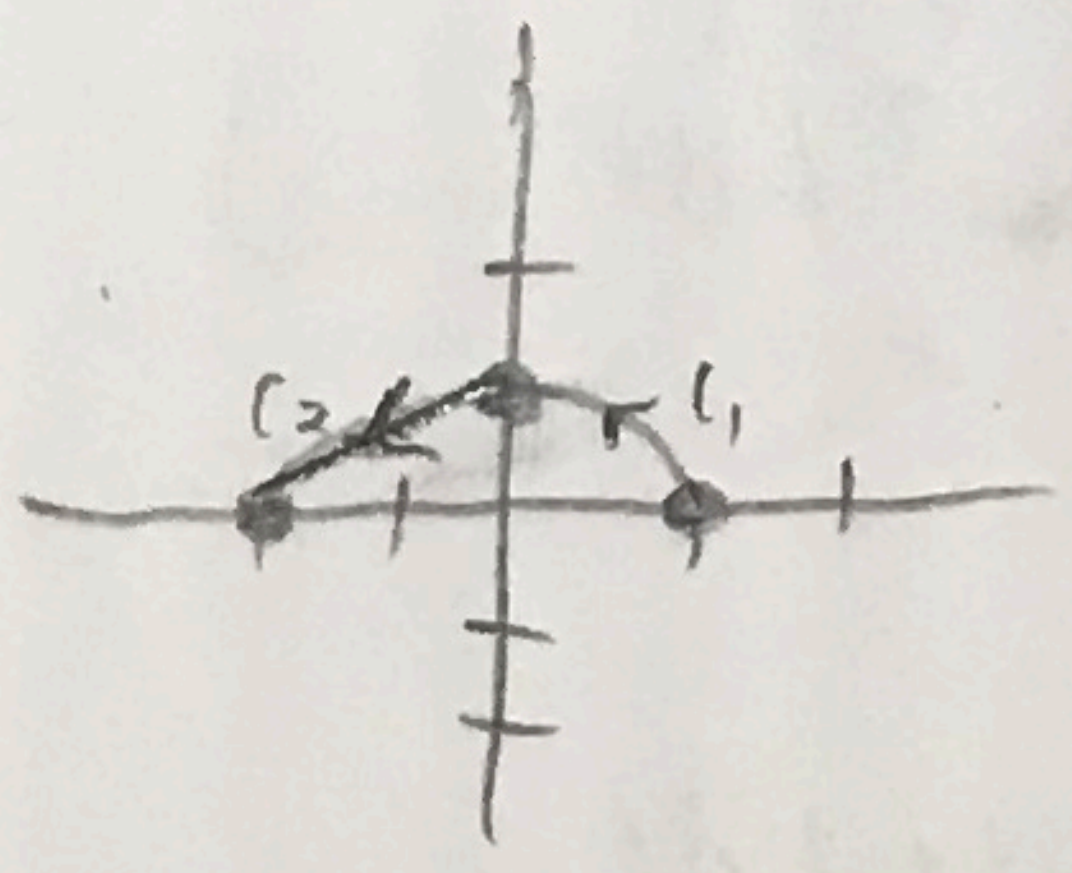
(a) (5 points) Show that the vector field $\mathbf{F} = \langle e^y, xe^y \rangle$ is conservative and evaluate $\int_C \mathbf{F}\, d\mathbf{r}$.

(5) (5 points) Evaluate $\int_C \langle e^y, xe^y - x \rangle\, d\mathbf{r}$

$(a_1, b_1) = (-2,0)$

$(a_0, b_0) : (0,1)$

$Q_y = P_x$



(a) Conservative v.f

$\text{curl}(F) = 0$

Show $\left(\begin{array}{l} F = \nabla f \\ \text{potential exists} \\ F = \nabla f \end{array} \right.$

$F = \langle \overset{F_1}{e^y}, \overset{F_2}{xe^y} \rangle$

potential function exists and is: $\boxed{f = xe^y}$ ✓

$e^y = f_x$ $\quad f = xe^y$

$xe^y = f_y$ $\quad f = xe^y$

$r(\tfrac{\pi}{2}) = (0,1)$

$r(0) = (1,0)$

$r(t)$ $C_1 : (\cos t, \sin t)$

$0 \le t \le \frac{\pi}{2}$

$r(t)$ $C_2 : (-2t, 1-t)$

$0 \le t \le 1$

$$\int_{C_1} F \cdot dr = f(r(\tfrac{\pi}{2})) - f(r(0)) = f(0,1) - f(1,0)$$

$$= (0 +)1 - = -1$$

$$\int_{C_1} F\,dr + \int_{C_2} F\,dr$$

$$\int_{C_2} F \cdot dr = f(r(1)) - f(r(0)) = f(-2,0) - f(0,1) = -2$$

$$\int_C F\,dr = \int_{C_1} F\,dr + \int_{C_2} F\,dr = -1 - 2 = \boxed{-3}$$

(b) $\int_C \langle e^y, xe^y - x \rangle\, dr$

$F : \langle e^y, xe^y - x \rangle = \langle e^y, xe^y \rangle + \langle 0, -x \rangle$

$$\underbrace{\int_C \langle e^y, xe^y \rangle\, dr}_{\text{solved above}} + \int_C \langle 0, -x \rangle\, dr$$

$C_1 : \; r(t) = (\overset{x}{\cos t}, \overset{y}{\sin t})$

$0 \le t \le \frac{\pi}{2}$

$r'(t) = (-\sin t, \cos t)$

$C_2 : \; r(t) = t(-2,0) + (1-t)(0,1)$

$= \boxed{\langle \overset{x}{-2t}, \overset{y}{1-t} \rangle \quad 0 \le t \le 1}$

$r'(t) = \langle -2, -1 \rangle$

$$\int_C F\,dr = \int_C F(r(t)) \cdot r'(t)\,dt$$

$$= \underline{-3} + \int_{C_1} (0,-x)\,dr + \int_{C_2} (0,-x)\,dr$$

$$-3 + \int_0^{\frac{\pi}{2}} (0, -\cos t) \cdot (*, \cos t)\,dt + \int_0^1 (0, 2t) \cdot (*, -1)\,dt$$

$$\int_0^1 t\,dt = \frac{t^2}{2}\Big|_0^1 = \frac{1}{2}$$

$$-3 + \int_0^{\frac{\pi}{2}} -\cos^2 t\,dt + \int_0^1 -2t\,dt$$

$$-2\int_0^1 t\,dt = -2(\tfrac{1}{2}) = -1$$

$$-3 - \int_0^{\frac{\pi}{2}} \cos^2 t\,dt - 2\int_0^1 t\,dt$$

On back

$\sin^2 = 1-\cos^2\theta$

$-3 - \underline{\qquad} - 1$

$$-\int_0^{\frac{\pi}{2}} \cos^2 t \, dt$$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$\cos 2\theta = \cos^2\theta - (1-\cos^2\theta)$

$\cos 2\theta = 2\cos^2\theta - 1$

$\frac{\cos 2\theta + 1}{2} = \cos^2\theta$

$$= -\frac{1}{2}\int_0^{\frac{\pi}{2}} \cos 2t + 1 \, dt$$

$$= -\frac{1}{2}\int_0^{\frac{\pi}{2}} \cos 2t \, dt - \frac{1}{2}\int_0^{\frac{\pi}{2}} 1 \, dt$$

$u = 2t \quad u(\frac{\pi}{2}) = \pi$

$u(0) = 0$

$du = 2dt$

$\frac{1}{2} du = dt$

$$-\frac{1}{4}\int_{u=0}^{\pi} \cos u \, du \qquad - \frac{1}{2} t \Big|_0^{\frac{\pi}{2}}$$

$$-\frac{1}{4} \sin u \Big|_0^{\pi} \qquad - \frac{\pi}{4}$$

$$\cancel{= 0}$$

$$= -\frac{\pi}{4}$$

$$\int_C \langle e^y, xe^y - x \rangle \, dr = -3 - \frac{\pi}{4} - 1$$

$$= \boxed{-4 - \frac{\pi}{4}}$$

PHYSICS 1A

Winter, 2017

Midterm 1

Dr. Coroniti

There are 100 points on the exam, and you have 50 minutes. To rec
show all you work and reasoning. No credit will be giv
"appear". The exam is closed not

Problem 5. (10 points) Let C be the closed (segmented) path with vertices $(1,0),(2,0),(0,2),(0,1)$, oriented counterclockwise. Evaluate $\oint_C \langle \underset{P}{xy+\sin(x^3)}, \underset{Q}{x^2+e^{y^2}} \rangle \, dr$

$y = 2-x$

$y = 1-x$

$x+y=2$

$x+y=1$

$1 \ge x+y \le 2$

need Jacobian

Greens: $\oint_C F\,dr = \iint_D Q_x - P_y \, dA$

$= \iint_D 2x - x \, dA = \iint_D x \, dA$

look at vertically simple

2

2 regions vertically simple

① $0 \le x \le 1$

$1-x \le y \le 2-x$

$\int_{x=0}^{1} \int_{y=1-x}^{2-x} x \, dy \, dy$

$\int_{x=0}^{1} \left(xy \Big|_{y=1-x}^{2-x} \right) dx$

$\int_{x=0}^{1} 2x-x^2-(x-x^2)\,dx$

$\int_{x=0}^{1} 2x-\cancel{x^2}-x+\cancel{x^2}\,dx$

$\int_{x=0}^{1} x\,dx = \frac{x^2}{2}\Big|_0^1$

$= \frac{1}{2}$

② $1 \le x \le 2$

$0 \le y \le 2-x$

$\int_{x=1}^{2} \int_{y=0}^{2-x} x \, dy \, dx$

$\int_{x=1}^{2} xy \Big|_{y=0}^{2-x} dx$

$\int_{x=1}^{2} 2x-x^2 \, dx = x^2 - \frac{x^3}{3} \Big|_1^2$

$= 4 - \frac{8}{3} - (1-\frac{1}{3})$

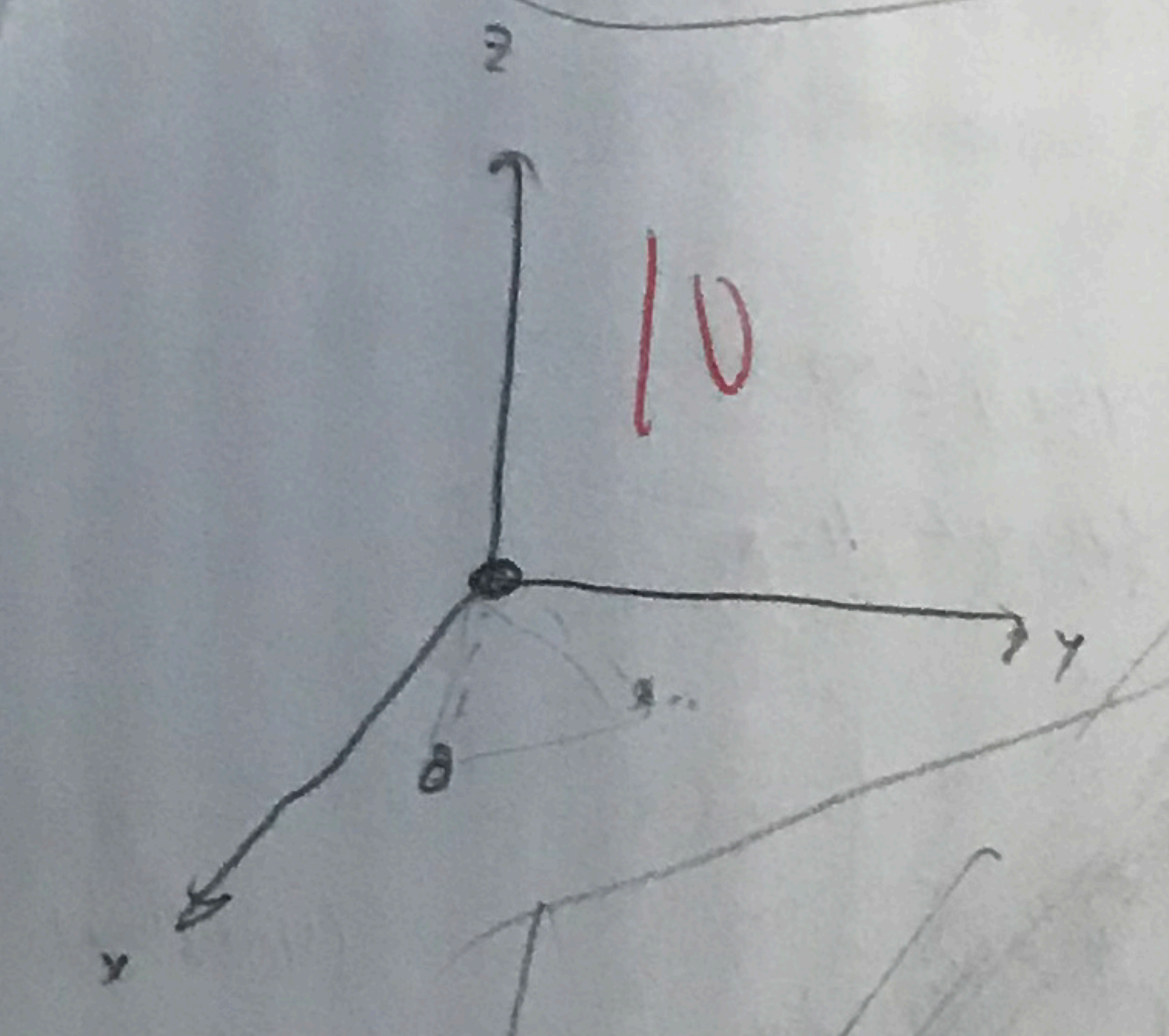
$3 - \frac{7}{3}$

$+ 3 - \frac{7}{3} = \frac{3}{6} + \frac{18}{6} - \frac{14}{6} = \boxed{\frac{7}{6}}$

Dr. Coroniti

oints on the exam, and you have 50 minutes. To receive full credit,
ork and reasoning. No credit will be given for answers that simply
ar". The exam is closed notes and closed book. You do not need calculators, so
nd all cell phones, away. If you need more space, use th

Problem 6. (15 points) Let S be the triangle surface with vertices $(0,0,0)$, $(3,1,1)$, $(1,3,1)$ oriented with upward normals. Evaluate the flux $\iint_S \langle -y, 4z, x\rangle \, d\mathbf{S}$



10

projection on x, y

closed surface ✓

~~Divergence thm~~

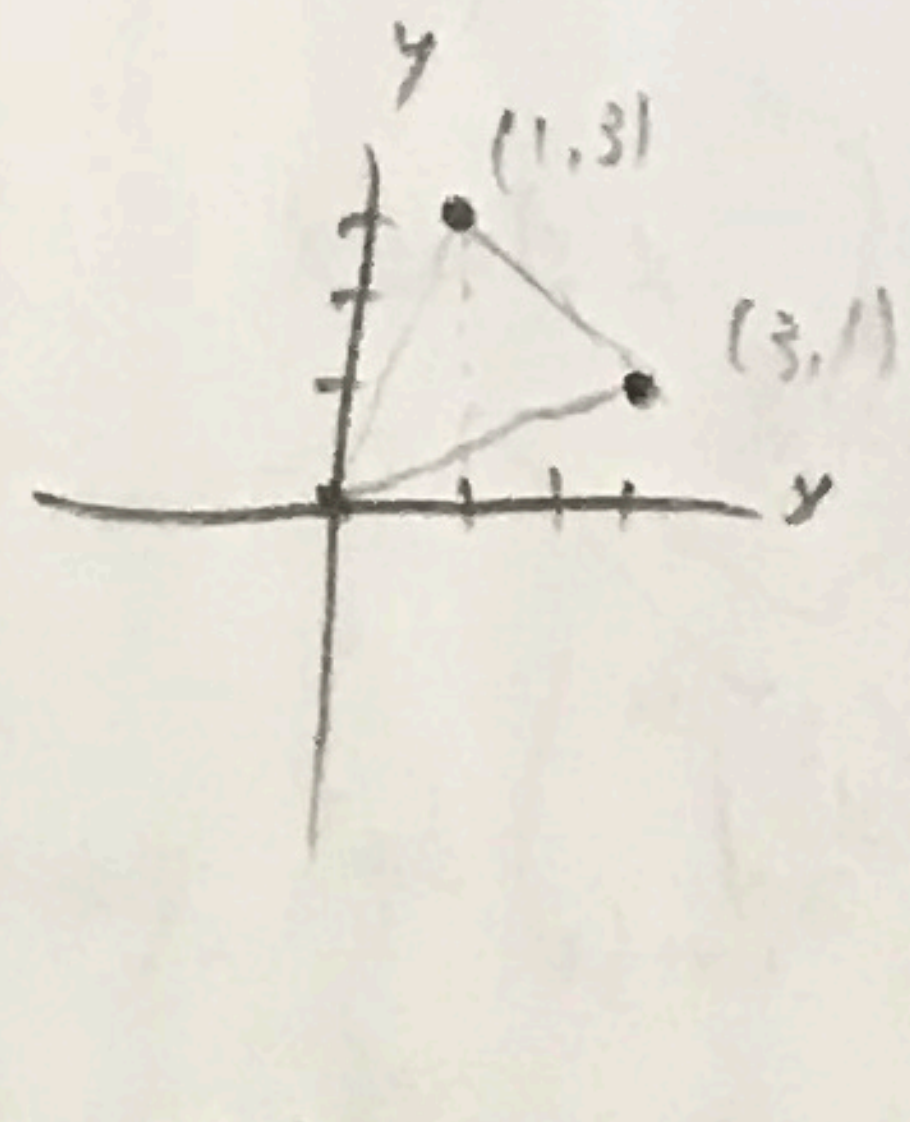
~~closed surface $\iint_S F \cdot dS = \iiint div(F) dV$~~

~~$F = \langle -y, 4z, x\rangle$~~

~~$div(F) = \nabla \cdot F = \langle 0 + 0 + 0\rangle =$~~

~~$\iiint 0 \, dV$~~

~~$= 0$~~



$x + y + z = 1$

2 regions

① $0 \le x \le 1$

$\frac{1}{3}x \le y \le 3x$

② $1 \le y \le 3$

$\frac{1}{3}x \le y \le 4 - x$

$0 \le z \le 1$

$\iint_S F \, dS = \iint_D F(G(x,y)) \cdot (G_x \times G_y) dA$

$G(x,y) = (x, y,$

surface parametrization

✓ $G(x,y) = (x, y, \frac{1}{4}(x+y))$ ✓

~~potential~~ plane equation for triangle

$\boxed{x + y - 4z = 0}$

$Ax + By + Cz = 1$

$-4z = -x - x$

$z = \frac{1}{4}x + \frac{1}{4}y$

$G(x,y) = (x, y, \frac{1}{4}(x+y)$

$G_x = (1, 0, \frac{1}{4})$

$G_y = (0, 1, \frac{1}{4})$

$$G_x \times G_y = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & \frac{1}{4} \\ 0 & 1 & \frac{1}{4} \end{vmatrix}$$

$G_x \times G_y = \langle +\frac{1}{4}, +\frac{1}{4}, 1\rangle$ ✗

$F = \langle -y, 4z, x\rangle$

$F(G(x,y)) = \langle -y, x+y, x\rangle$

$\iint_S F dS = \iint_D F(G(x,y)) \cdot (G_x \times G_y) dA = \iint_D \langle -y, x+y, x\rangle \cdot \langle \frac{1}{4}, \frac{1}{4}, 1\rangle dA = \iint_D -\frac{1}{4}y + \frac{1}{4}x + \frac{1}{4}y + \frac{1}{4}x \, dA$

$= \frac{1}{2}\iint_D x \, dA$ ✗ : On back:

$\frac{1}{2}\iint_D x\,dA$

Break D into 2 vertically simple regions

① $0 \le x \le 1$

$\frac{1}{3}x \le y \le 3x$

add 2 regions

② $1 \le x \le 3$

$\frac{1}{3}x \le y \le 4-x$

$\frac{1}{2}\int_{x=0}^{1}\int_{y=\frac{1}{3}x}^{3x} x\,dy\,dx$

$(\frac{1}{3}x)^2 = \frac{1}{9}x^2$

$\frac{1}{2}\int_{x=0}^{1}\left(\frac{x^2}{2}\Big|_{y=\frac{1}{3}x}^{3x}\right)dx$

$\frac{1}{2}\int_{x=0}^{1}\frac{9x^2-\frac{1}{9}x^2}{2}\,dx$

$\frac{1}{4}\int_{x=0}^{1} 9x^2-\frac{1}{9}x^2\,dx$

$\frac{1}{4}\left(3x^3-\frac{1}{27}x^3\Big|_0^1\right)$

$\frac{1}{4}(3-\frac{1}{27})$

$\frac{1}{2}\int_{x=1}^{3}\int_{y=\frac{1}{3}x}^{4-x} x\,dy\,dx$

$(4-x)(4-x)$

$x^2-8x+16$

$\frac{1}{2}\int_{x=1}^{3}\left(\frac{x^2}{2}\Big|_{\frac{1}{3}x}^{4-x}\right)dx$

$\frac{1}{2}\int_{x=1}^{3}\frac{x^2-8x+16-\frac{1}{9}x^2}{2}\,dx$

$\frac{1}{4}\int_1^3 x^2-8x+16-\frac{1}{9}x^2\,dx$

$\frac{1}{4}\left(\frac{x^3}{3}-4x^2+16x-\frac{1}{27}x^3\Big|_1^3\right)$

$\frac{1}{4}\left((9-36+48-1)-(\frac{1}{3}-4+16-\frac{1}{27})\right)$

$\frac{1}{4}(20-\frac{1}{3}+4-16-\frac{1}{27})$

$\frac{1}{4}(8-\frac{1}{3}-\frac{1}{27})$

add

$\frac{1}{4}(3-\frac{1}{27}+8-\frac{1}{3}-\frac{1}{27})$

$\frac{1}{4}(11-\frac{1}{3}-\frac{2}{27})$

$\boxed{\frac{1}{4}(11-\frac{11}{27})}$

Winter, 2017

There are 100 points on the exam,
show all you work and reasoning. No cr
"appear". The exam is closed notes an
nd all cell phones,

Problem 7. (15 points) Let G
oriented outwards. (S is not
(a) (7 points) Find a vector
(b) (8 points) Evaluate

outward normal

F = LP

curl (

(a)

$-Qz$

P

inter, 2017

HYSICS 1A

Midterm 1

Dr. Coroniti

There are 100 points on the exam, and you have 50 minutes. To receive full credit, show all you work and reasoning. No credit will be given for answers that simply "appear". The exam is closed notes and closed book. You do not need calculators, so and all cell phones, away. If you need more space, use the ba

$z = \sqrt{1-x^2-\frac{y^2}{4}}$

Problem 7. (15 points) Let $\mathbf{G} = \langle 1, y, -z\rangle$ and S be the half of the ellipsoid $x^2+\frac{y^2}{4}+z^2=1$ for $x \geq 0$, oriented outwards. (S is not closed.)

(a) (7 points) Find a vector potential $\mathbf{F}$ so that $\mathbf{G} = \text{curl}(\mathbf{F})$.

(b) (8 points) Evaluate $\iint_S \mathbf{G}\, d\mathbf{S}$ using Stokes' theorem.

$x = \cos t$

outward normal ✓

$x \quad y \quad z$

$G(\theta,\phi) = (\cos\theta\sin\phi,\ 2\sin\theta\sin\phi,\ \cos\phi)$

$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$

$0 \leq \phi \leq \frac{\pi}{2}$

$F = \langle P, Q, 0\rangle$

(a) $\text{curl}(F) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & 0 \end{vmatrix} = \langle -Q_z, P_z, Q_x - P_y\rangle$

$-Q_z = 1 \qquad Q = -z$

$P_z = y \qquad P = yz$

$Q_x - P_y = -z \qquad 0 - z = -z$ ✓

$F = \langle P, Q, 0\rangle$

$\boxed{F = \langle yz, -z, 0\rangle}$

$\text{curl}(F) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & -z & 0 \end{vmatrix}$

$\text{curl}(F) = G$

$G = (1, y, -z)$ ✓

Stokes'

(b) $\iint_S G\, dS = \iint_S \text{curl}(F)\, dS = \oint_C F\, dr$

$\oint_C F\, dr = \int_t F(r(t)) \cdot r'(t)\, dt$

$P =$

$G(x,y) = \langle x, y, \sqrt{1-x^2-\frac{y^2}{4}}\rangle$

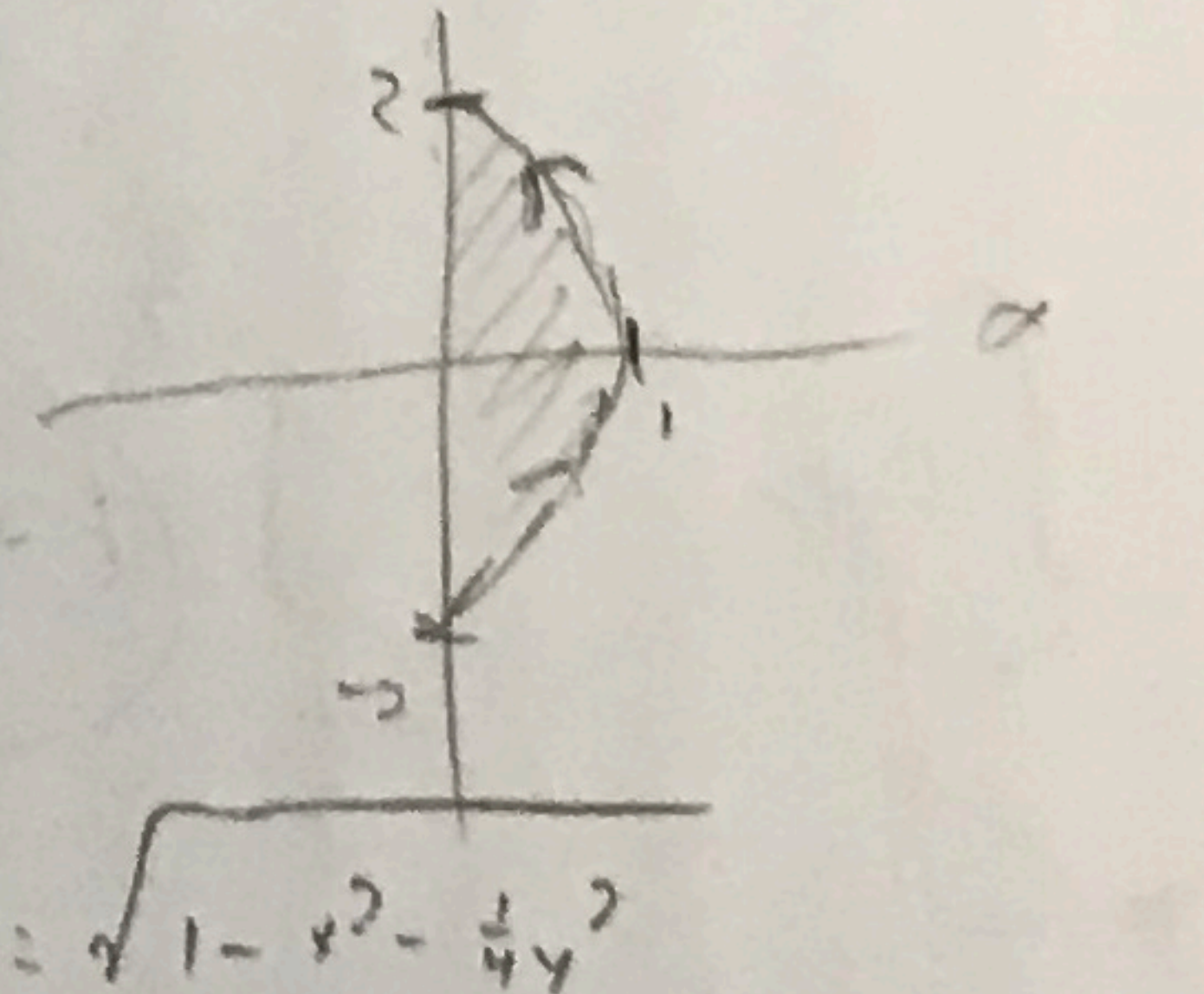
projection of boundary

$x = \cos\theta$

$y = 2\sin\theta$

$z = \sqrt{1-\cos^2\theta-\sin^2\theta}$

$z = \sqrt{1-r^2}$



$z = \sqrt{1-x^2-\frac{1}{4}y^2}$

$r(\phi) = (\cos\theta, 2\sin\theta,$

$-\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}$

~~$\langle P, 0, Q\rangle$~~

~~$\langle yz, 0, y\rangle$~~

~~$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & 0 & Q \end{vmatrix} = \langle Q_y, P_z - Q_x, -P_y\rangle$~~

~~$Q_y = 1 \qquad Q = y$~~

~~$P_z - Q_x = y$~~

~~$-P_y = -z \qquad P = yz \qquad (yz, 0, y)$~~

$G(x,y) = \langle x, y, \sqrt{1-x^2-\frac{y^2}{4}}\rangle$

~~$G(r,\theta) = \langle r\cos\theta, r\sin\theta, \sqrt{1-r^2\cos^2\theta-\frac{1}{4}r^2\sin^2\theta}$~~

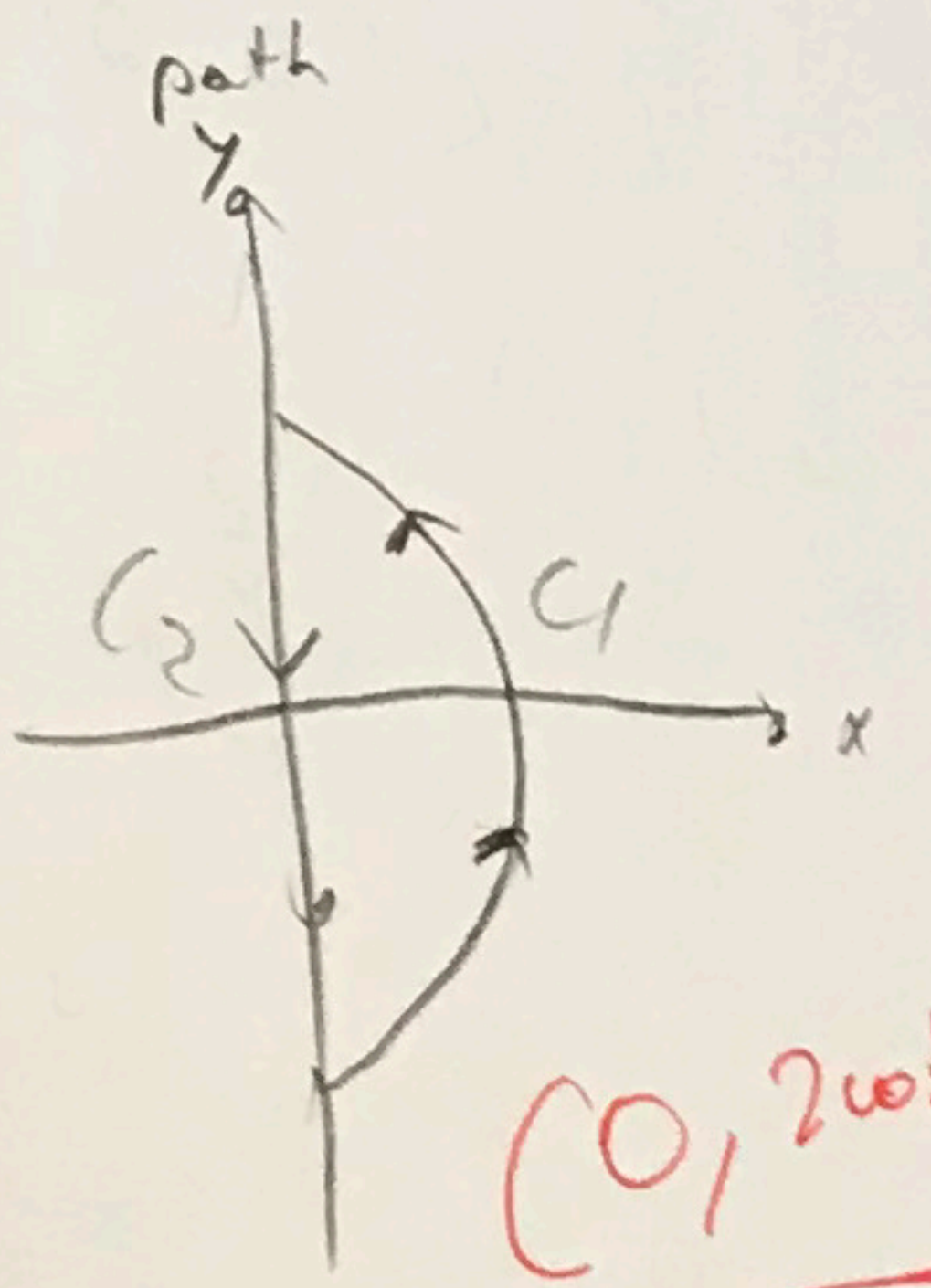
$x \quad y$

$G(\theta,\phi) = \langle \cos\theta\sin\phi, 2\sin\theta\sin\phi, \sqrt{1-\cos^2\theta\sin^2\phi-\sin^2\theta\sin^2\phi}$

On back

$G(\theta,\phi) = \left(\cos\theta\sin\phi, 2\sin\theta\sin\phi, \sqrt{1-\cancel{\cos^2\theta\sin^2\phi} - \sin^2\theta\sin^2\phi}\right)$

$G(\theta,\phi) = (\overset{x}{\cos\theta\sin\phi}, \overset{y}{2\sin\theta\sin\phi}, \sqrt{1-\sin^2\phi})$



$F = \langle yz, -z, 0)$

$C_2: \oint_2^{-2} (t,-1,0)\cdot(0,1,0)$

$\oint_2^{-2} -1\,dt = -t\Big|_2^{-2}$

$-(-2-2) = 4$

$\bigcirc 4$

$(0, 2\cos t, \sin t)$

$C_1: r(t) = (\overset{x}{\cos t}, \overset{y}{2\sin t}, \overset{z}{1})$

$-\frac{\pi}{2} \le t \le \frac{\pi}{2}$

$r'(t) = \langle \underset{x}{-\sin t}, \underset{y}{2\cos t}, \underset{z}{0})$

$C_2: r(t) = (0, t, 1)$

$2 \le t \le -2$

$r'(t) = (0,1,0)$

-4

$x^2 + \frac{y^2}{4} + z^2 = 1$

$x = \cos t$

$y = 2\sin t$

$\cos^2 t + \sin^2 t + z^2 = 1$

$z^2 = 1$

$z = 1$

$\oint_{C_1} F\,dr = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \langle \overset{yz}{2\sin t}, \overset{-z}{-1}, \overset{0}{0}) \cdot \overbrace{\langle -\sin t, 2\cos t, 0\rangle}^{r'(t)}$

$= \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -2\sin^2 t - 2\cos t\,dt = -2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 t + \cos t\,dt$

$-2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 t\,dt - 2\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos t\,dt$

$-\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} 1-\cos 2t\,dt$

$-\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dt + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos 2t\,dt$

$-\frac{\pi}{2} - \frac{\pi}{2} = -\pi +$

$-\left(2\sin\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\right)$

$-(2(1-(-1))$

-4

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos 2t\,dt$

$\frac{1}{2}\int_{-\pi}^{\pi} \cos u\,du$

$\frac{1}{2}\sin u\Big|_{-\pi}^{\pi} = 0$

$\cancel{\cos 2\theta = \cos^2\theta - \sin^2\theta}$

$\sin^2 t = \frac{1-\cos 2\theta}{2}$

$u = 2t$

$du = 2dt$

$\frac{1}{2}du = dt$

$u(\frac{\pi}{2}) = \pi$

$u(-\frac{\pi}{2}) = -\pi$

$-\pi + \cancel{0} - 4$

$\cancel{-\pi - 4}$

$-\pi - 4 + 4$

$= \boxed{-\pi}$

PHYSICS 1A

Midterm 1

Winter, 2017

Dr. Coroniti

There are 100 points on the exam, and you have 50 minutes. To receive full credit, show all you work and reasoning. No credit will be given for answers that simply "appear". The exam is closed notes and closed book. You do not need calculators, so
nd all cell phones, away. If you need more space, use the back

---

Problem 8. (15 points) Let W be the region enclosed by the planes $z = x$, $z = -2x$, $x + y + z = 1$, and $y = 0$. Let S be the (closed) boundary surface of W. Evaluate the flux $\iint_S \langle 2x, y+z, x-z\rangle \, dS$

P O R

Divergence thm

10

$$\iint_S F\,dS = \iiint_W \mathrm{div}(F)\,dV$$

$-2x \le z \le x$

$F = (2x, y+z, x-z)$

$\mathrm{div}(F) = (2+1-1) = 2$

$\nabla \cdot F = 2$

$$\iint_S F\,dS = 2\iiint_W dV$$

$2(\text{volume}(W))$

$$\text{volume}(W) = \int_{x=0}^{1}\int_{y=0}^{1-x}\int_{z=-2y}^{x} dz\,dy\,dx$$

?

$$= \int_{x=0}^{1}\int_{y=0}^{1-x} 3x\,dy\,dx$$

$$\int_{x=0}^{1} 3xy\Big|_0^{y=1-x} dx$$

$$= \int_{x=0}^{1} 3x - 3x^2\,dx$$

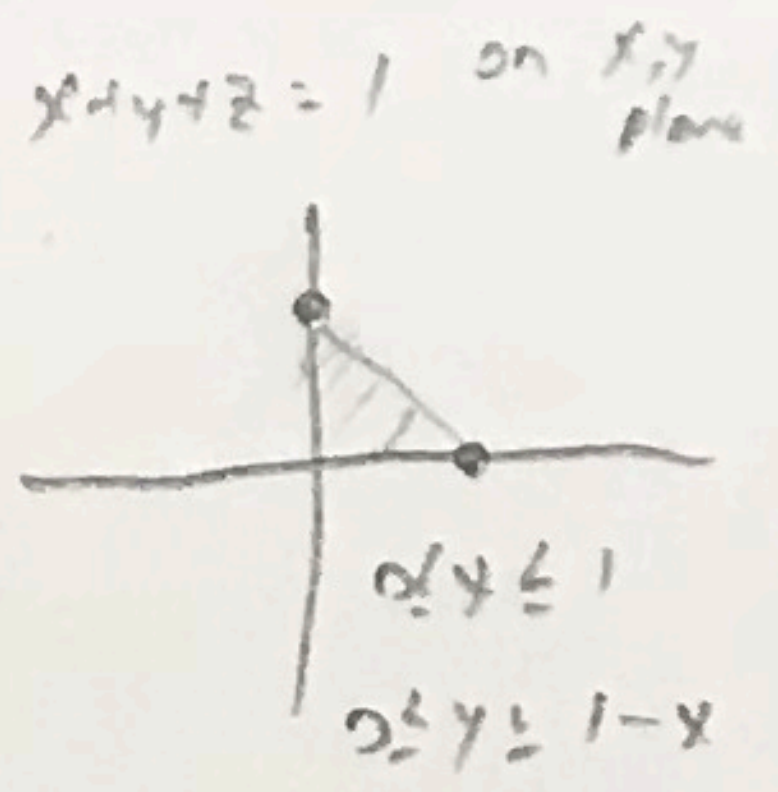
~~volume of ( ) = $\frac{abc}{6}$~~

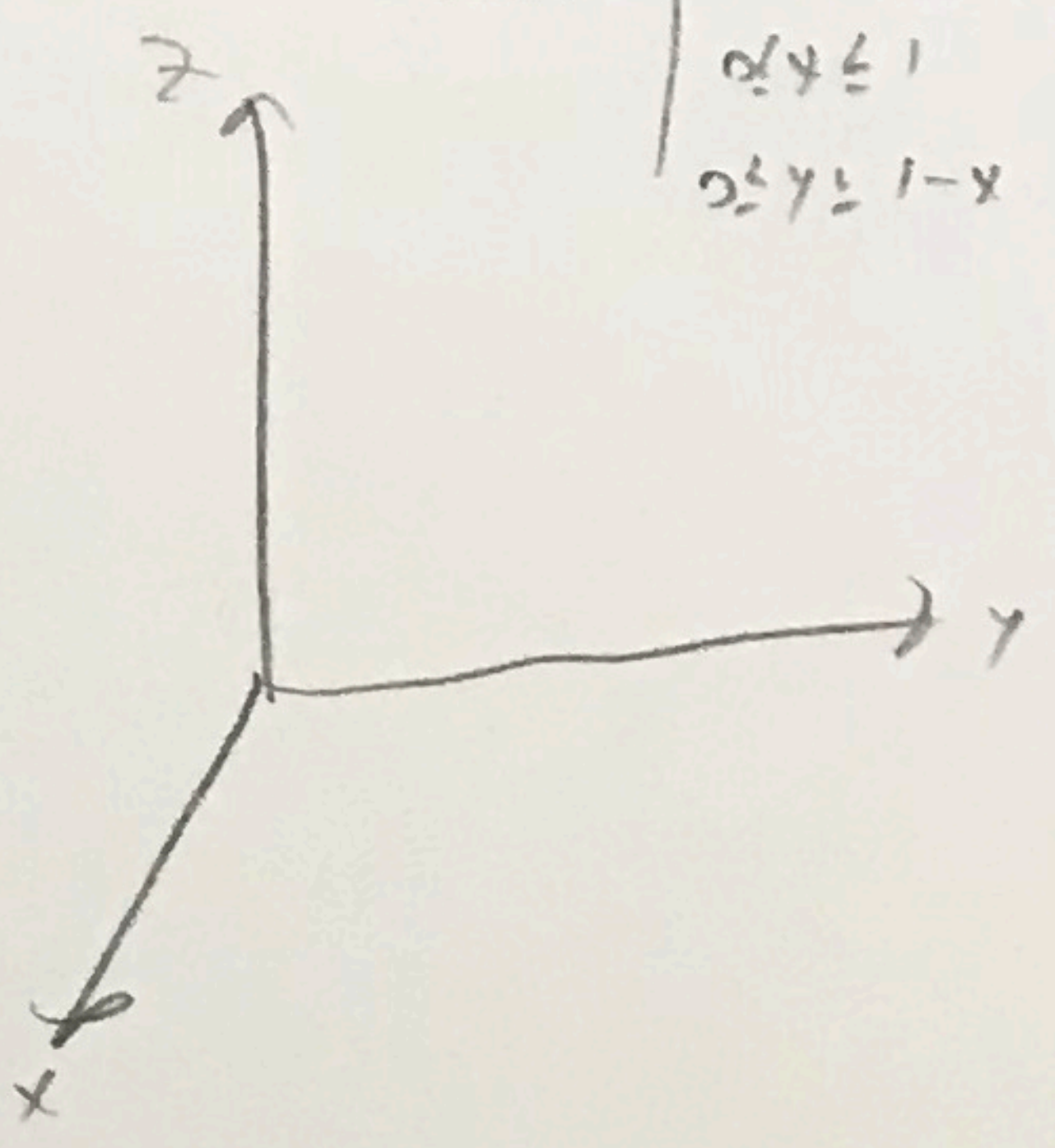
~~$0 \le x \le 1$~~

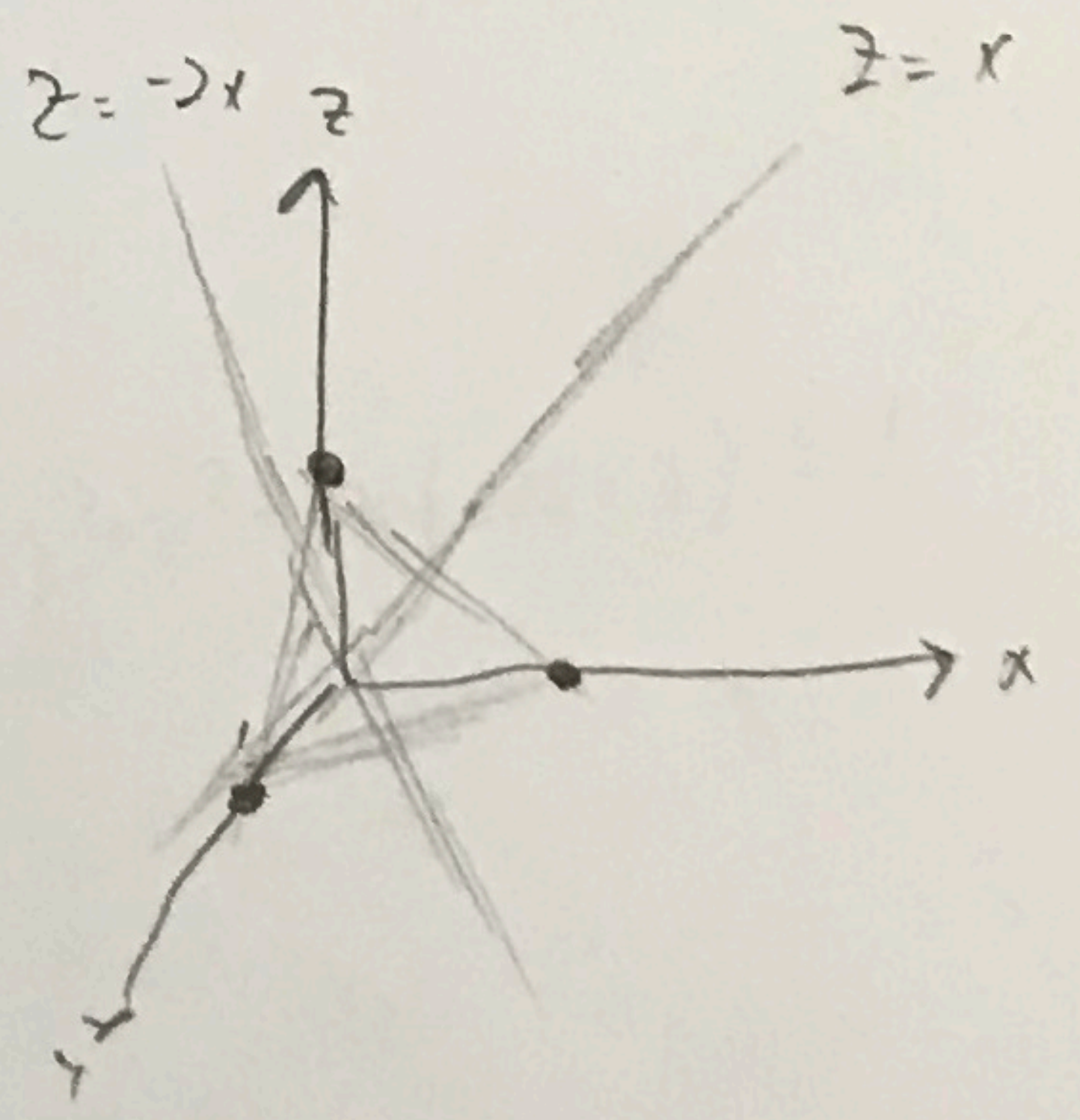
~~$0 < y \le 1-x$~~

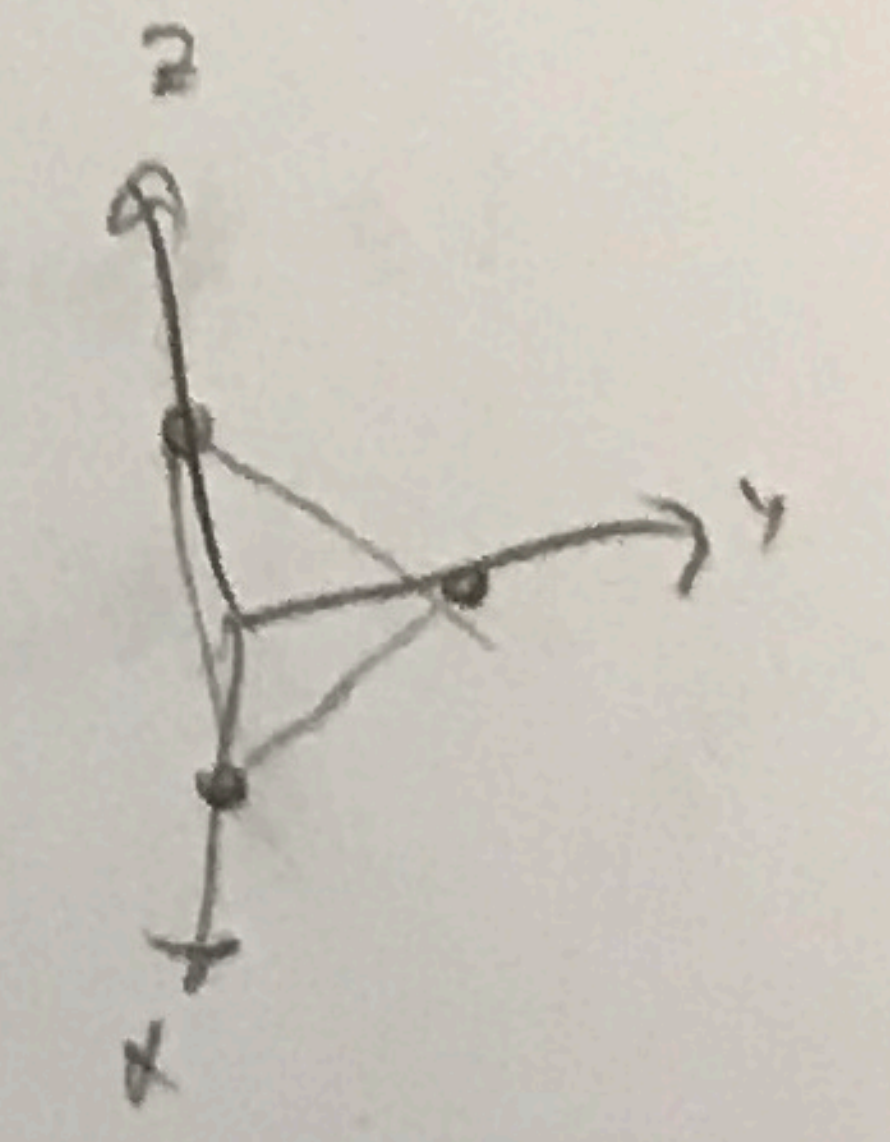
~~$\int_{x=0}^{1}\int_{y=0}^{1-x}\int_{z=0}^{1} dz\,dy\,dx = \int_0^1\int_0^{1-x} dy\,dx = \int_0^1 1-x\,dx = x - \frac{x^2}{2}\Big|_0^1 = 1-\frac{1}{2} = \frac{1}{2}$~~

$$\int_{x=0}^{1} 3x - 3x^2\,dx = \frac{3x^2}{2} - x^3\Big|_0^1 = \frac{3}{2} - 1 = \boxed{\frac{1}{2}}$$









ublic Affairs Building 2270

MECH&AE 105A

Winter, 2017

PHYSICS 1A

Midterm 1

Dr. Coroniti

There are 100 points on the exam, and you have 50 minutes. To receive full
show all you work and reasoning. No credit will be given for answers th
The exam is closed notes and closed book. You do n
phones. away. If you ne

Problem 9. (15 points) Let W be the region containing all points (x, y, z) satisfying

$$x^2 + y^2 + z^2 + xy + xz \leq 1.$$

Find the volume of W.

$z^2 + xz = 1 - x^2 - y^2 - xy$

$x^2 + y^2 + z^2 \leq 1 - xy - xz$

$\mathrm{vol}(W) = \iiint_W dV$

$(x+y)^2 + (z-x)^2$

$x^2 + 2xy + y^2 + z^2 - 2xz + x^2$

$(x+y)^2 = x^2 + y^2 + 2xy$

$(x-y)^2 = x^2 + y^2 - 2xy$

$(x+z)^2 = x^2 + z^2 + 2xz$

$(x-z)^2 = x^2 + z^2 - 2xz$

$-(x+z)^2 = -x^2 z^2 - 2xz$

$x = r\cos\theta$

$y = r\sin\theta$

$z = \rho\cos\phi$

$x = \rho\cos\theta\sin\phi$

$y = \rho\sin\theta\sin\phi$

$z = \rho\cos\phi$

$r = \rho\sin\theta$

$y^2 + z^2 + x(x+y+z) \leq 1$

$x^2 + y^2 = r^2$

$x^2+y^2 \quad z^2 \quad xy \quad xz$

$(\rho\sin\theta)^2 + (\rho\cos\theta)^2 + (\rho^2\sin\theta\cos\theta\sin^2\phi) + \rho^2\cos\theta\sin\phi\cos\phi \leq 1$

$\rho^2(\sin^2\theta + \cos^2\theta) + \rho^2\sin\theta\cos\theta\sin^2\phi + \rho^2\cos\theta\sin\phi\cos\phi \leq 1$

$\rho^2 + \rho^2\sin\theta\cos\theta\sin^2\phi + \rho^2\cos\theta\sin\phi\cos\phi \leq 1$