MATH 32B Midterm II Practice Feb. 24, 2020

STUDENT NAME: SOLUTIONS

STUDENT ID NUMBER: ____________

DISCUSSION SECTION NUMBER: ____________

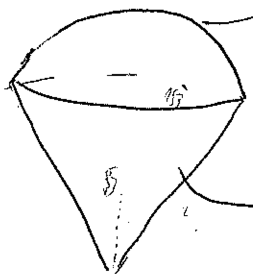
Directions

Answer each question in the space provided. Please write clearly and legibly. Show all of your work—your work must both justify and clearly identify your final answer. No books, notes or calculators are allowed. You must simplify results of function evaluations when it is possible to do so.

For instructor use only

Page	Points	Score
2	8	
3	10	
4	12	
5	12	
6	8	
Total:	50	

1. [8 pts] Find the volume of the ice cream cone bounded above by the sphere of radius 6, and bounded below by the cone $x^2 + y^2 = 3z^2$.



$\rho = 6$

$\rho^2 \sin^2\phi = 3\rho^2\cos^2\phi$

$\tan\phi = \sqrt{3}$

$\phi = \frac{\pi}{3}$

So $\displaystyle\int_{\theta=0}^{2\pi}\int_{\phi=0}^{\pi/3}\int_{\rho=0}^{6} \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$

$= \displaystyle\int_{\theta=0}^{2\pi}\int_{\phi=0}^{\pi/3} \frac{1}{3}6^3 \sin\phi \, d\phi \, d\theta$

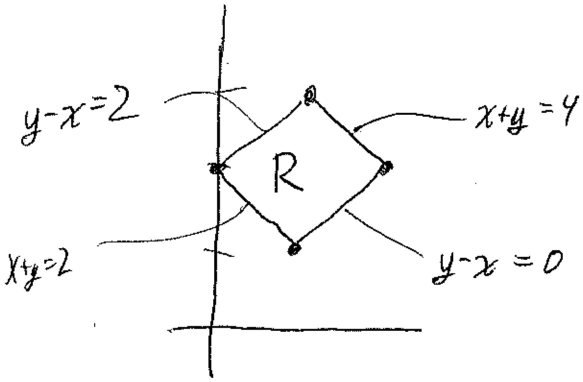
$= \displaystyle\int_{\theta=0}^{2\pi} -72\cos\phi \Big|_{\phi=0}^{\pi/3} d\theta$

$= \displaystyle\int_{\theta=0}^{2\pi} 36 \, d\theta = 72\pi$

2. [10 pts] Evaluate

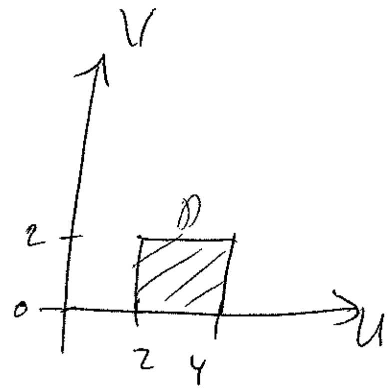
$$\iint_R \frac{x-y}{x+y}\,dA$$

where R is the square with vertices (0,2), (1,1), (2,2), and (1,3). HINT: Use change of variables!



$$\xrightarrow{F}$$

$$\begin{cases} u = x+y \\ v = -x+y \end{cases}$$



$$dF = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$|dF| = |2| = 2$$

$$\text{So } \iint_R \frac{x-y}{x+y}\,dA = \int_{v=0}^{2}\int_{u=2}^{4} \frac{-v}{u}\cdot\frac{1}{2}\cdot du\,dv$$

$$= \frac{1}{2}\int_{v=0}^{2} -v\ln u \Big|_{u=2}^{4}\,dv$$

$$= -\frac{1}{2}\int_{u=0}^{2} v(\ln 4 - \ln 2)\,dv$$

$$= +\frac{\ln 2 - \ln 4}{2}\cdot\frac{1}{2}v^2\Big|_0^2$$

$$= \ln 2 - \ln 4$$

3. [6 pts] Let $f(x, y) = x^2 + 3y$, and let L be the line segment from $(3,0)$ to $(0,4)$. Find $\int_L f(x,y)ds$.

$\vec{r}(t) = (1-t)\langle 3,0\rangle + t\langle 0,4\rangle$

$= \langle 3-3t, 4t\rangle$ FROM $t=0$ TO $t=1$

$\vec{r}'(t) = \langle -3, 4\rangle$ $|\vec{r}'(t)| = 5$

$\int_L f\,ds = \int_{t=0}^{1} \left((3-3t)^2 + 3(4t)\right)\cdot 5\,dt$

$= 5\int_{t=0}^{1} (9 - 18t + 9t^2 + 12t)\,dt$

$= 15\int_{t=0}^{1} (3 - 2t + 3t^2)\,dt$

$= 15\left[3t - t^2 + t^3 \Big|_{t=0}^{1}\right]$

$= 45$

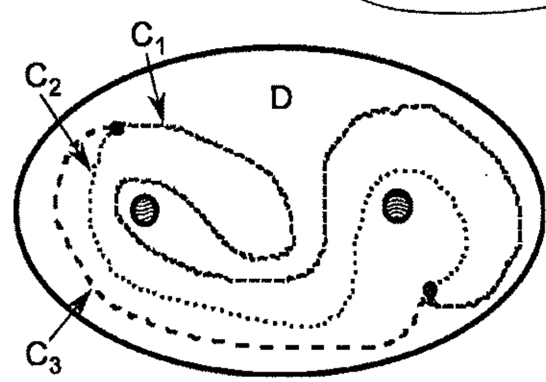
4. TRUE/FALSE (circle your answer, no justification needed)

(a) [3 pts] If curl $\vec{\mathbf{F}} = 0$ throughout D, then $\int_{C_1} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_{C_2} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ in the picture below.

TRUE *FALSE*

(b) [3 pts] If curl $\vec{\mathbf{F}} = 0$ throughout D, then $\int_{C_2} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_{C_3} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ in the picture below.

TRUE *FALSE*



5. Let $\vec{\mathbf{A}}(x,y,z) = \langle e^z, e^z, e^z \rangle$ and let $\vec{\mathbf{B}}(x,y,z) = \langle e^z, e^z, e^z(1+x+y+z) \rangle$.

(a) [6 pts] Is $\vec{\mathbf{A}}$ conservative or not? If not, justify. If so, find a potential function.

$$\text{CURL } \hat{A} = \vec{\nabla} \times \langle e^z, e^z, e^z \rangle$$

$$= \langle e^z, \text{OTHERS} \rangle \neq \vec{0}, \text{ SO } \underline{\text{NOT}} \text{ CONSERVATIVE}$$

(b) [6 pts] Is $\vec{\mathbf{B}}$ conservative or not? If not, justify. If so, find a potential function.

CURL $\vec{B} = \vec{0}$, SO FIND POTENTIAL:

$$\frac{\partial f}{\partial x} = B_1 = e^z \implies f = e^z x + g(y,z)$$

$$\frac{\partial f}{\partial y} = \frac{\partial g}{\partial y} = B_2 = e^z \implies g = ye^z + h(z), \text{ so } f = xe^z + ye^z + h(z)$$

$$\frac{\partial f}{\partial z} = xe^z + ye^z + h'(z) = B_3 = e^z + xe^z + ye^z + ze^z$$

$$\implies h'(z) = e^z + ze^z$$

$$\implies h(z) = ze^z + C$$

$$\text{So } f(x,y,z) = e^z(x+y+z)$$

(SETTING C=0)

(c) [3 pts] Let C_1 be the unit circle centered at the origin. Choose one of the following to compute:

$$\oint_{C_1} \vec{\mathbf{A}} \cdot d\vec{\mathbf{r}} \quad \text{OR} \quad \oint_{C_1} \vec{\mathbf{B}} \cdot d\vec{\mathbf{r}} = 0$$

BECAUSE $\vec{B}$ CONSERVATIVE

(DOMAIN IS ALL OF $\mathbb{R}^3$, SIMPLY CONNECTED)

(d) [5 pts] Let C_2 be the line segment from point $P = (0, 1, -1)$ to $Q = (5, 4, 0)$. Choose one of the following to compute:

$$\int_{C_2} \vec{\mathbf{A}} \cdot d\vec{\mathbf{r}} \quad \text{OR} \quad \int_{C_2} \vec{\mathbf{B}} \cdot d\vec{\mathbf{r}}$$

$$\int_{C_2} \vec{B} \cdot d\vec{r} = f(Q) - f(P)$$

$$= e^0(5+4+0) - \cancel{e^{-1}(0+1+-1)}^{\,0}$$

$$= 9$$