$u=1$
$v=1$

Problem 4. (4)

Let S be a surface given by the parametric equation $\mathbf{r}(u,v) = u\mathbf{i}+v\mathbf{j}+(u^2-2v^2)\mathbf{k}$. Find the equation of the tangent plane of S at the point $(1,1,-1)$.

$\vec{r}_u = 1\vec{i} + 0\vec{j} + 2u\,k$

$\vec{r}_v = 0\vec{i} + 1\vec{j} - 4v\,k$

$$\vec{r}_u \times \vec{r}_v = \left|\begin{matrix} \vec{i} & \vec{j} & k \\ 1 & 0 & 2u \\ 0 & 1 & -4v \end{matrix}\right| \begin{matrix} \vec{i} & \vec{j} \\ 1 & 0 \\ 0 & 1 \end{matrix}$$

$0k - 2u\vec{i} + 4v\vec{j} + 0\vec{i} + 0\vec{j} + 1\vec{k}$

$= -2u\vec{i} + 4v\vec{j} + 1k$

$\vec{n} = \langle -2, 4, 1 \rangle$

4

Equation of the Tangent Plane

$-2(x-1) + 4(y-1) + 1(z-(-1)) = 0$

$$\boxed{-2(x-1) + 4(y-1) + (z+1) = 0}$$

Problem 5. (4)

Evaluate the surface integral $\int\int_S (x^2+y^2)dS$ where S is the sphere given by the equation $x^2+y^2+z^2=a^2$.

$x: a\sin\phi\cos\theta$

$y: a\sin\phi\sin\theta$

$z: a\cos\theta$

$$\int\int_S (x^2+y^2)ds$$

$$\int\int (a^2\sin^2\phi\cos^2\theta + a^2\sin^2\phi\sin^2\theta)\, a^2\sin\phi\, d\phi\, d\theta$$

$$\int\int (a^2\sin^2\phi)(a^2\sin\phi)$$

$$\int_0^{2\pi}\int_0^{\pi} (a^3\sin^3\phi\, d\phi\, d\theta$$

$$= 2\pi a^3\left(\frac{-\cos\phi^4}{4}\right)$$

$$2\pi a^3\left(\frac{1}{4}+\frac{1}{4}\right)$$

$$2\pi a^3\, 2/4 = \boxed{a^3\pi}$$

2

$$\int_0^{2\pi}\int_0^{\pi} (a^2\sin^2\phi)\, a^2\sin\phi$$

$$= a^4(2\pi)\int_0^{\pi} (\sin\phi)^3 d\phi$$

1	4
2	2.5
3	4
4	4
5	2
T	16.5

$\sin^2 t + \cos t^2 = 1$

$1-\cos^2$

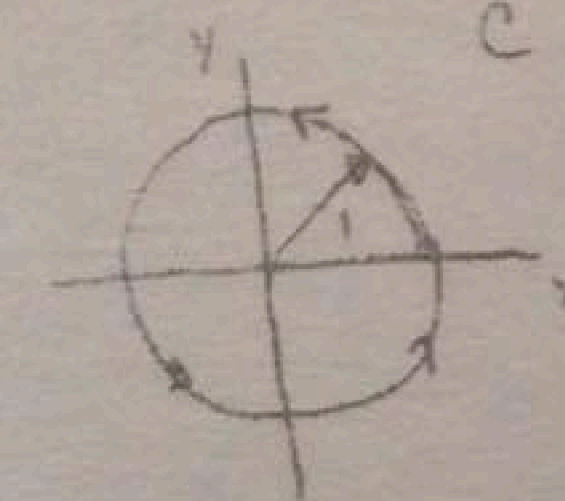
Problem 2. (4)

Evaluate the following line integral

$$\int_C 2y\,dx - x\,dy$$

by two different methods:(a) compute it directly; (b) use the Green theorem to compute it. Here C is the unit circle $x^2 + y^2 = 1$ with the counter clockwise orientation.

C



directly

(a) $0 \le t \le 2\pi$

$x: \cos t \Rightarrow -\sin t\,dt$

$y: \sin t \Rightarrow \cos t\,dt$

$$\int_0^{2\pi}\left[2\sin t(-\sin t)dt - \cos t(\cos t)dt\right]$$

$$\int_0^{2\pi}\left[-(2)(\sin^2 t) - \cos t^2\right]dt$$

$$\int_0^{2\pi}\left[-2\sin t^2 - (1-\sin t^2)\right]dt$$

$$\int_0^{2\pi}(-1-\sin t^2)\,dt$$

$$\int_0^{2\pi}\left[-t-\left[\frac{t}{2}+\frac{t}{4}\sin t^2\right]\right]_0^{2\pi}$$

$= -\pi$

Answer

$$\int_0^{2\pi}\left[-t-\frac{t}{2}+\frac{2}{4}\sin 2t\right]_0^{2\pi}$$

$= -3\pi$

Green's theorem

$$\int_0^{2\pi}\int_0^1\left(\frac{d(x)}{dx}-\frac{d(2y)}{dy}\right)r\,dr\,d\theta$$

$$\int_0^{2\pi}\int_0^1(1-2)\,r\,dr\,d\theta$$

$$\frac{-r^2}{2}\Big|_0^1(2\pi)$$

$= -1\pi$

$(-1-2)r$

$-3r(2$

$-\frac{3r^2}{2}$

$= -3\pi$

1
2
3
4
5
T

MATH 32B Midterm II, Fall 2010

Name: [redacted]

TA's Name and Section Number: Ricketson - 1A

Problem 1. (4)

Find the line integral

$$\int_C x^3 dx + y^2 dy + z dz,$$

where C is the line segment connecting the point $P = (0,0,1)$ and $Q = (1,2,4)$, and C is oriented by the direction from P to Q.

R_0 R_1

Parameters

$0<t<1$

$x: (1-t)0 + t = t$

$y: (1-t)0 + 2t = 2t$

$z: (1-t)1 + 4t = 1 - t + 4t = 1 + 3t$

$$\int_0^1 \left[t^3(1dt) + 4t^2(2dt) + (1+3t)(3dt) \right]$$

$$= \int_0^1 (t^3 + 8t^2 + 3 + 9t)\, dt$$

$$= \left[\frac{t^4}{4} + \frac{8t^3}{3} + 3t + \frac{9t^2}{2} \right]_0^1$$

$$= \frac{1}{4} + \frac{8}{3} + 3 + \frac{9}{2}$$

$$= \frac{3}{12} + \frac{32}{12} + \frac{36}{12} + \frac{54}{12}$$

$$\boxed{= \frac{125}{12}}$$

3
32
36
54
125

4

4

Problem 3. (4)

Let $F = (2x+3y)\mathbf{i} + (3x-10y)\mathbf{j} + 4z\mathbf{k}$ defined on $\mathbb{R}^3$.
(i) Decide whether or not F is conservative.
(ii) If F is conservative, find the potential function f, such that $F = \nabla f$.
(iii) Compute the line integral $\int_C F \cdot dr$, where C is given by the parametric equation $x = t$, $y = t^2$, $z = t^3$ $0 \le t \le 1$ with the orientation given by the parametrization.

$0 \le t \le 1$

$x = t$
$y = t^2$
$z = t^3$

(i) curl $\vec{F}$

$$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ d/dx & d/dy & d/dz \\ (2x+3y) & (3x-10y) & (4z) \end{vmatrix}$$

$-3\vec{k} - 0\vec{i} - 0\vec{j} + 0\vec{i} + 0\vec{j} + 3\vec{k}$

$= 0\vec{i} + 0\vec{j} + 0\vec{k}$

curl $\vec{F} = 0$, thus conservative

ii) $\vec{F} = (2x+3y)\vec{i} + 3x - 10y\,\vec{j} + 4z\vec{k}$

$f(x,y,z) = x^2 + 3xy + g(y,z)$

$f_y = 0 + 3x + g'(y,z)$

$g'_y = -10y$

$g_y = -5y^2 + h(z)$

$f = x^2 + 3xy - 5y^2 + h(z)$

$f_z = 0 + 0 + 0\ (h'z)$

$h'(z) = 4z$

$h(z) = 2z^2 + k$

Potential function

$f(x,y,z) = x^2 + 3xy - 5y^2 + 2z^2 + k$

k → can be any constant

(iii) $\int_C \vec{F} \cdot d\vec{r}$

$= \int_C \nabla f \cdot d\vec{r} = f(1) - f(0)$

$f(0,0,0) = 0$

$f(1,1,1) = 1 + 3 - 5 + 2 = 1$

$= \int_C \vec{F} \cdot d\vec{r} = 1$

$\vec{r}_u =$
$\vec{r}_v =$
$\vec{r}_u \times \vec{r}_v$
$= -$
$\vec{n} = \langle -$
Equ
-2
-2