2

1	3
2	0
3	1
4	4
5	3.[illegible]
T	11.[illegible]

MATH 32B Midterm I, Fall 2010

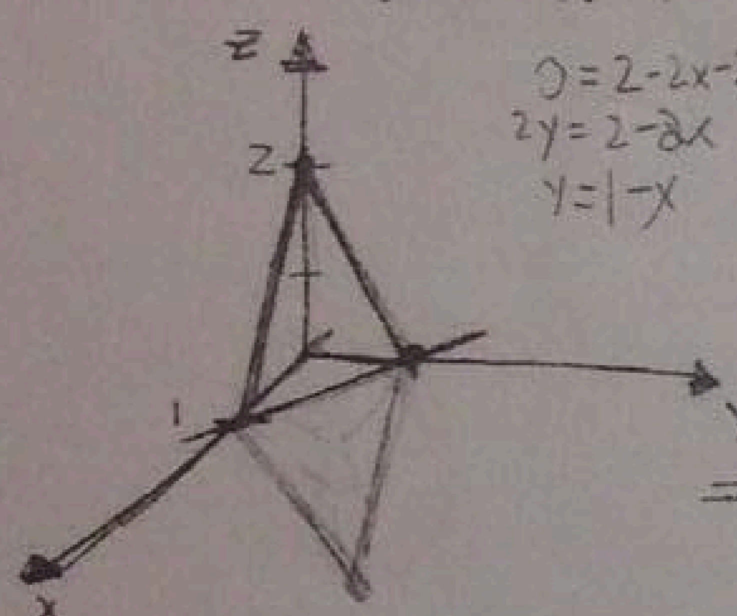
Na[redacted]'s Name and Section Number: [redacted]

Problem 1. (4)

Find the triple integral $\int\int\int_E z\,dx\,dy\,dz$. Here E is the solid bounded by the four planes $x = 0$, $y = 0$, $z = 0$ and $2x + 2y + z = 2$.

$z = 2 - 2x - 2y$

$0 = 2 - 2x - 2y$
$2y = 2 - 2x$
$y = 1 - x$



$$\int_0^1 \int_0^{1-x} \int_0^{2-2x-2y} z \; dz\,dy\,dx$$

$$= \int_0^2 \int_0^{1-\frac{z}{2}} \int_0^{1-y} z \, dx\, dy\, dz$$

3

$$= \int_0^2 \int_0^{1-z/2} z\, x\Big|_0^{1-y} dy\,dz$$

$$\int_0^2 \int_0^{1-z/2} (1-y) z \, dy\, dz$$

$$\int_0^2 \int_0^{1-z/2} z - yz \, dy\,dz$$

$$\int_0^2 \left(yz - \frac{y^2}{2} z\right)\Big|_0^{1-z/2} dz$$

$$\left(1-\frac{z}{2}\right)z - \left(\frac{z}{2} - \frac{z^2}{2} + \frac{z^3}{8}\right) dz$$

$$z - \cancel{\frac{z^2}{2}} - \frac{z}{2} + \cancel{\frac{z^2}{2}} - \frac{z^3}{8}\, dz$$

$$\int_0^2 \frac{z}{2} - \frac{z^3}{8}\, dz$$

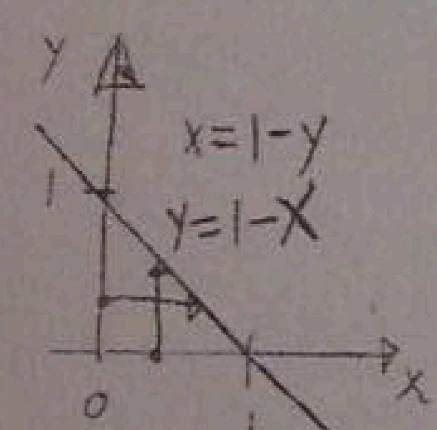
$$= \frac{z^2}{4} - \frac{z^4}{32}\Big|_0^2$$

$1 - 16/32 = 1/2$

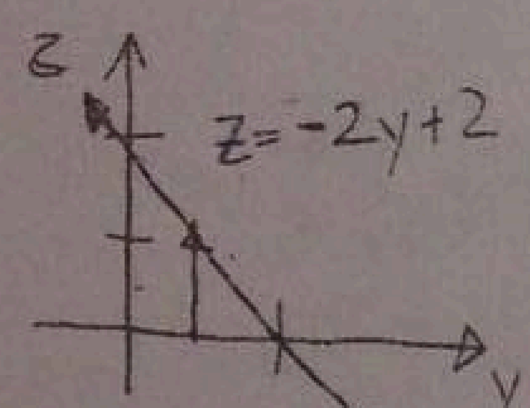
$\boxed{= \frac{1}{2}}$ X

Right integral, wrong answer.

$(1-z/2)(1-z/2)$
$1 - \frac{2z}{2} + \frac{z^2}{4}$
$1 - z + z^2/4$



$x = 1 - y$
$y = 1 - x$



$z = -2y + 2$

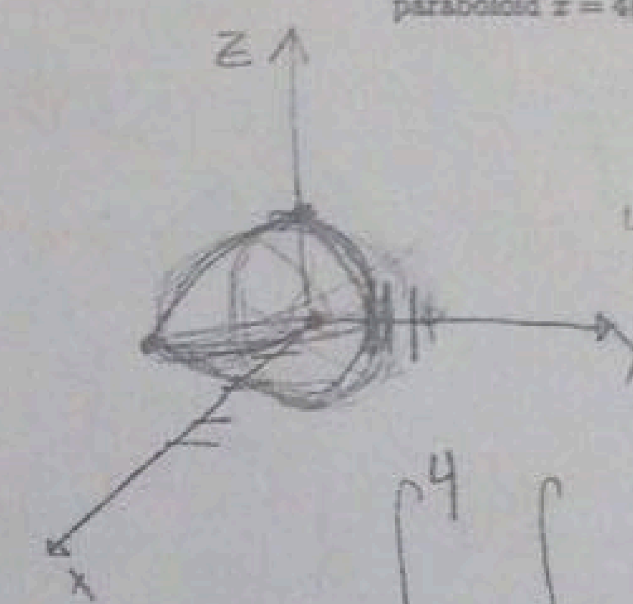
224

$y = \frac{2 - z}{2}$
$= 1 - z/2$

Problem 2. (4)

Find the triple integral $\int\int\int_E x\,dV$, where E is the region bounded by the paraboloid $x = 4(y^2 + z^2)$ and the plane $x = 4$.

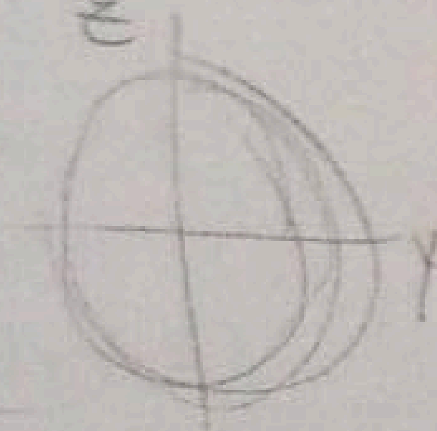
$x = 4(y^2 + z^2)$
$x = 4y^2 + 4z^2$
$4z^2 = x - 4y^2$
$z = \pm\sqrt{\frac{1}{4}x - 4y^2}$



$$\int_0^4 \int \int_0 x \; dz\,dy\,dx$$

0

$4 = 4(y^2 + z^2)$
$1 = y^2 + z^2$

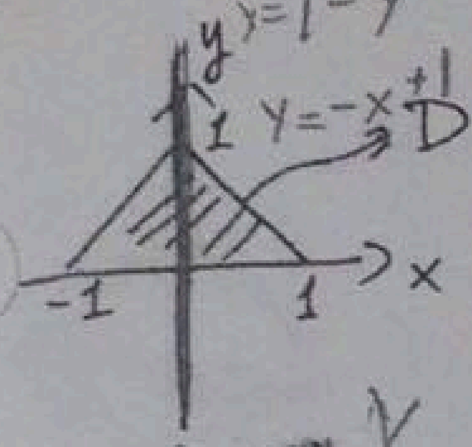


$$m = \iint \rho(x,y)\,dA$$
$$M_x = \iint y\,\rho(x,y)\,dA$$
$$M_y = \iint x\,\rho(x,y)\,dA$$

using symmetry

Problem 3. (4)

Compute the center of mass of D. Here the density fuction $\rho(x, y) = y$.



$$m = 2\int_0^1\int_0^{-x+1} y\,dy\,dx$$

$$= 2\int_0^1 \frac{y^2}{2}\Big|_0^{-x+1} dx \qquad (-x+1)(-x+1) \quad x^2-2x+1$$

$$= \int_0^1 x^2-2x+1\,dx$$

$$\frac{x^3}{3} - x^2 + x\Big|_0^1$$

$$= 1-1+1 = 1$$

— by symmetry & homogenuity

$$M_y = 2\int_0^1\int_0^{-x+1} xy\,dy\,dx$$

$$= 2\int_0^1 x(x^2-2x+1)\,dx$$

$$= x^3 - 2x^2 + x$$

$$= \left(\frac{x^4}{4} - \frac{2x^3}{3} + \frac{x^2}{2}\right)2$$

$$= \left(1 - \frac{2}{3} + \frac{1}{2}\right)2$$

$$= \left(\frac{3}{3} - \frac{2}{3} + \frac{1}{2}\right)2$$

$$= \left(\frac{1}{3} + \frac{1}{2}\right)$$

$$= \frac{2}{6} + \frac{3}{6} = \frac{10}{6}$$

$$M_x = 2\int_0^1\int_0^{1-y} y^2\,dx\,dy$$

$$= 2\int_0^1 (1-y)y^2\,dy$$

$$y^2 - y^3$$

$$\frac{y^3}{3} - \frac{y^4}{4}$$

$$-\frac{1}{4} = \frac{1}{2}$$

$$(\bar{x}, \bar{y}) = \left(\frac{10}{6}, 1\right)$$

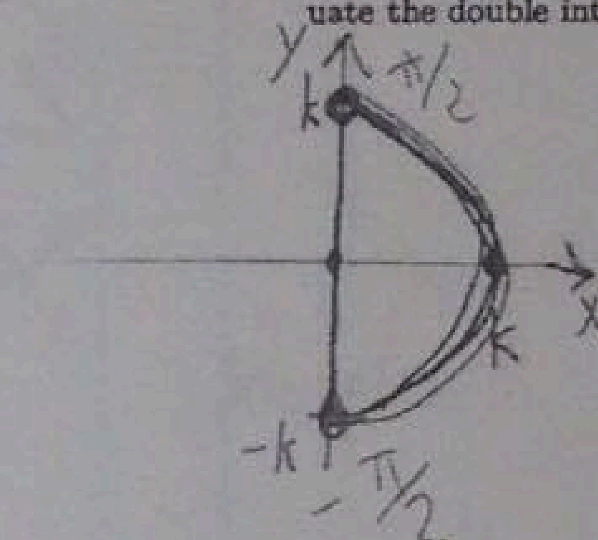
$$= (0, 1$$

Problem 4. (4)

Find the iterated integral

$$\int_0^k \int_{-\sqrt{k^2-x^2}}^{\sqrt{k^2-x^2}} \sin(x^2+y^2)\,dy\,dx.$$

Hint: Convert the iterated integal into a proper double integral and then evaluate the double integral.



$$\sin(r^2)$$

4

$$\int_{-\pi/2}^{\pi/2}\int_0^k \sin(r^2)\,r\,dr\,d\theta$$

$$\int_{-\pi/2}^{\pi/2}\left(-\frac{\cos(r^2)}{2}\Big|_0^k\right)d\theta$$

$$\int_{-\pi/2}^{\pi/2}\left(-\frac{\cos k^2}{2} + \frac{1}{2}\right)d\theta$$

$$\pi\,\frac{1}{2}\left(1 - \cos k^2\right)$$

$$\int_{-\pi/2}^{\pi/2} d\theta$$

$$\theta\Big|_{-\pi/2}^{\pi/2}$$

$$\frac{\pi}{2} + \frac{\pi}{2}$$

$$\frac{2\pi}{2}$$

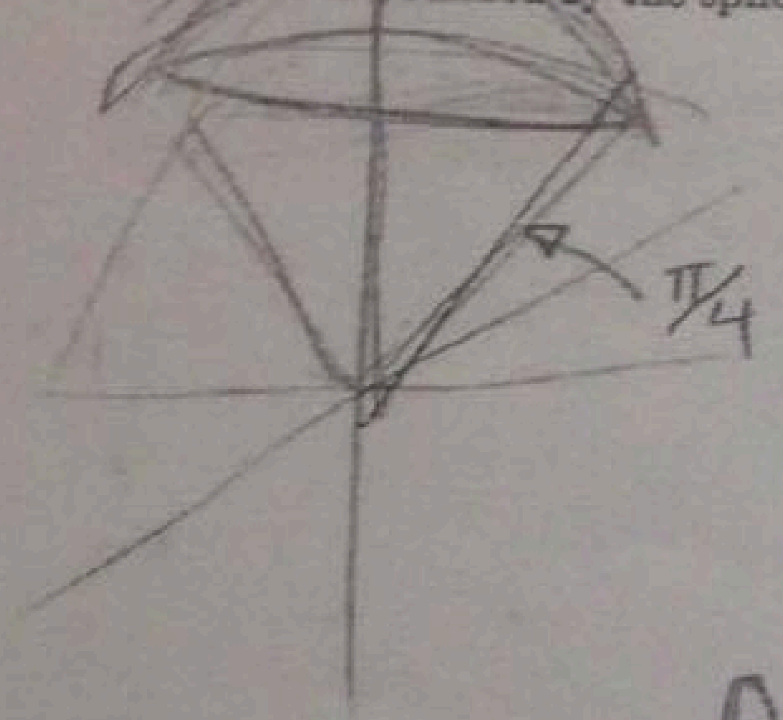
Period
Orbita
Lewis
Valenc
Bond
Symm
Molec
HOMC
Lattice
Allotro
Haloge
Band th
Metals,
Superc
Magneti
Close-p
Transiti
Types o

Problem 5. (4)

Find

$$\iiint_E z \, dx dy dz,$$

where E is bounded by the sphere $x^2 + y^2 + z^2 = z$ and the cone $z = \sqrt{x^2 + y^2}$.



$$\rho^2 = \rho \cos\phi$$

$$\rho = \cos\phi$$

$$\int_0^{2\pi}\int_0^{\pi/4}\int_0^{\cos\phi} \rho \cos\phi \, \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= 2\pi \int_0^{\pi/4} \left(\frac{\rho^4}{4} \cos\phi \sin\phi \right)\Big|_0^{\cos\phi} d\phi$$

$$= \frac{\pi}{2} \int_0^{\pi/4} \cos^5\phi \sin\phi \, d\phi$$

$$=$$

3.5