Math 32B - Lecture 1
Fall 2021
Midterm 1
10/20/2021

Name: ____________________

UID: ____________________

Time Limit: 50 mins

Version A

This exam contains 8 pages (including this cover page) and 3 problems. There are a total of 25 points available.

Check to see if any pages are missing. Enter your name and UID at the top of this page.

You are allowed one handwritten side of US letter paper with notes. You may **not** use any other notes, books, calculators, or other assistance.

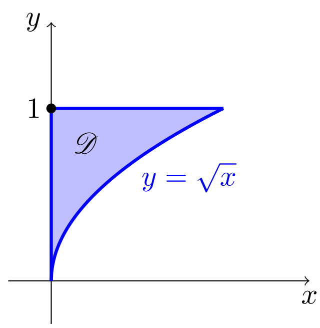
Please **switch off your cell phone** and place it in your bag or pocket for the duration of the test.

- Attempt all questions.
- Write your solutions clearly, in full English sentences, using units where appropriate.
- You may write on both sides of each page.
- You may use scratch paper if required.

1. (7 points) Compute

$$\iint_{\mathscr{D}} e^{y^3}\, dA,$$

where $\mathscr{D}$ is the region in the following sketch:



Solution: The region $\mathscr{D}$ can be written as a horizontally simple region,

$$\mathscr{D} = \left\{0 \le y \le 1 \quad , \quad 0 \le x \le y^2\right\}.$$

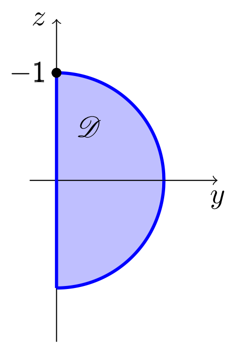
As a consequence,

$$\begin{aligned}\iint_{\mathscr{D}} e^{y^3}\, dA &= \int_0^1 \int_0^{y^2} e^{y^3}\, dx\, dy \\ &= \int_0^1 y^2 e^{y^3}\, dy \\ &= \tfrac{1}{3}(e-1).\end{aligned}$$

2. (9 points) Let $\mathscr{W}$ be bounded by $x = 0$, $y = 0$, $y = 1 - z^2$, and $x = 2$. Find

$$\iiint_{\mathscr{W}} x\, dV.$$

Solution: Observe that the projection of $\mathscr{W}$ onto the (y, z)-plane is the region bounded by $y = 0$ and $y = 1 - z^2$:



We can then write the region $\mathscr{W}$ in the form

$$\mathscr{W} = \left\{0 \leq x \leq 2\ ,\ 0 \leq y \leq 1 - z^2\ ,\ -1 \leq z \leq 1\right\}.$$

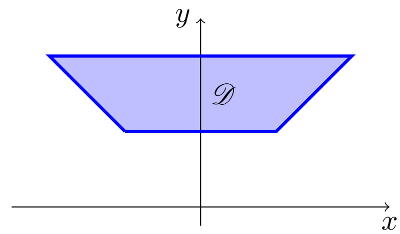
As a consequence,

$$\begin{aligned}\iiint_{\mathscr{W}} x\, dV &= \int_0^2 \int_{-1}^1 \int_0^{1-z^2} x\, dy\, dz\, dx \\ &= \int_0^2 \int_{-1}^1 x - xz^2\, dz\, dx \\ &= \int_0^2 \tfrac{4}{3} x\, dx \\ &= \tfrac{8}{3}.\end{aligned}$$

3. (9 points) Let $\mathscr{D}$ be the trapezium with corners $(-1,1)$, $(-2,2)$, $(2,2)$, $(1,1)$. Find

$$\iint_{\mathscr{D}} \left(x^2+y^2\right)^{-\frac{3}{2}}\, dA.$$

Solution: Let us first sketch the region $\mathscr{D}$:



Based on the integrand, let us convert to polar coordinates. In this case the line $y = 1$ becomes $r = \frac{1}{\sin\theta}$ and the line $y = 2$ becomes $r = \frac{2}{\sin\theta}$. As a consequence,

$$\begin{aligned}\iint_{\mathscr{D}} \left(x^2+y^2\right)^{-\frac{3}{2}}\, dA &= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \int_{\frac{1}{\sin\theta}}^{\frac{2}{\sin\theta}} \frac{1}{r^2}\, dr\, d\theta \\ &= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \frac{1}{2}\sin\theta\, d\theta \\ &= \frac{1}{\sqrt{2}}.\end{aligned}$$