Math 32B - Lecture 1 **Name:** ______________________
Fall 2018 **TA Section:** ______________________
Midterm 1
10/15/2018

Time Limit: 50 Minutes **Version** A

This exam contains 10 pages (including this cover page) and 4 problems. There are a total of 50 points available.

Check to see if any pages are missing. Enter your name and TA Section at the top of this page.

You may **not** use your books, notes or a calculator on this exam.

Please **switch off your cell phone** and place it in your bag or pocket for the duration of the test.

- Attempt all questions.
- Write your solutions clearly, in full English sentences, using units where appropriate.
- You may use scratch paper if required.

The majority of points on each problem will be for setting up the integrals correctly. At least one point on each problem will be for clearly explaining your solution, as on the homeworks.

1. (8 points) You are given the following values of a continuous function $f(x, y)$:

$y \backslash x$	1	1.5	2	2.5	3
2	3	3.5	4	4.5	5
1	2	2.5	3	3.5	4
0	1	1.5	2	2.5	3

Using a regular partition and the **upper-left** vertices of the subrectangles as sample points, approximate the integral $\int_1^3 \int_0^2 f(x, y)\, dydx$ by computing the Riemann sum $S_{4,2}$

TAKE THE PARTITION TO BE:

$x_0 = 1,\ x_1 = 1.5,\ x_2 = 2,\ x_3 = 2.5,\ x_4 = 3$

$y_0 = 0,\ y_1 = 1,\ y_2 = 2$

THEN $\Delta x_i = x_i - x_{i-1} = \frac{1}{2}$ FOR $i = 1,2,3,4$

AND $\Delta y_j = y_j - y_{j-1} = 1$ FOR $j = 1,2$

USING THE UPPER LEFT VERTICES AS SAMPLE POINTS, WE HAVE:

$P_{11} = 2,\ P_{21} = 2.5,\ P_{31} = 3,\ P_{41} = 3.5,$

$P_{12} = 3,\ P_{22} = 3.5,\ P_{32} = 4,\ P_{42} = 4.5$

THEN

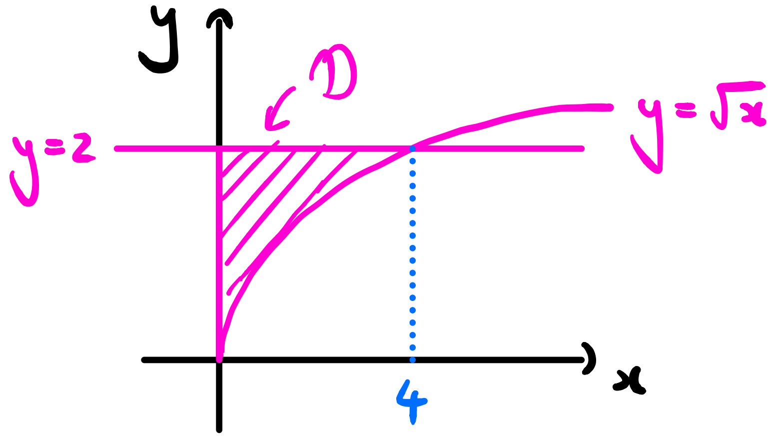
$$\int_1^3 \int_0^2 f(x,y)\, dydx \approx S_{4,2} = \sum_{i=1}^{4} \sum_{j=1}^{2} f(P_{ij}) \Delta y_j \Delta x_i$$

$$= (2 + 2.5 + 3 + 3.5 + 3 + 3.5 + 4 + 4.5) \cdot \frac{1}{2}$$

$$= 13.$$

2. (13 points) Find $\int_0^4 \int_{\sqrt{x}}^2 \sin(y^3)\,dydx$. *(Hint: You might wish to switch the order of integration.)*

LET D BE THE REGION



BY FUBINI'S THEOREM,

$$\int_0^4 \int_{\sqrt{x}}^2 \sin(y^3)\,dy\,dx$$

$$= \iint_D \sin(y^3)\,dA$$

$$= \int_0^2 \int_0^{y^2} \sin(y^3)\,dx\,dy$$

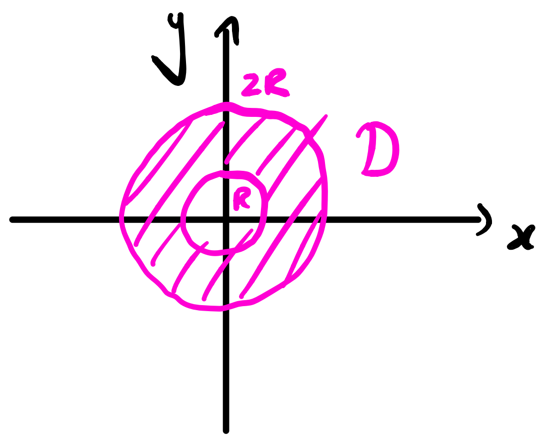
$$= \int_0^2 y^2 \sin(y^3)\,dy$$

$$= \left[-\frac{1}{3}\cos(y^3) \right]_{y=0}^{y=2}$$

$$= \frac{1}{3}\left(1 - \cos(8)\right).$$

3. (13 points) For $R > 0$ let $\mathcal{D} = \{(x,y) : R^2 \le x^2 + y^2 \le 4R^2\}$ and $f(x,y) = \dfrac{1}{x^2+y^2}$. Show that the value of $\displaystyle\iint_{\mathcal{D}} f(x,y)\,dA$ does not depend on the value of R.

FIRST SKETCH THE DOMAIN $\mathcal{D}$:



IN POLAR COORDINATES, $\mathcal{D} = \{0 \le \theta < 2\pi,\ R \le r \le 2R\}$.

THEN,

$$\iint_{\mathcal{D}} f(x,y)\,dA = \int_0^{2\pi}\int_R^{2R} \frac{1}{r^2}\, r\,dr\,d\theta$$

$$= \int_0^{2\pi} \Big[\ln r\Big]_{r=R}^{r=2R} d\theta$$

$$= \int_0^{2\pi} \ln(2R) - \ln(R)\ d\theta$$

$$= 2\pi \ln(2R) - 2\pi \ln(R)$$

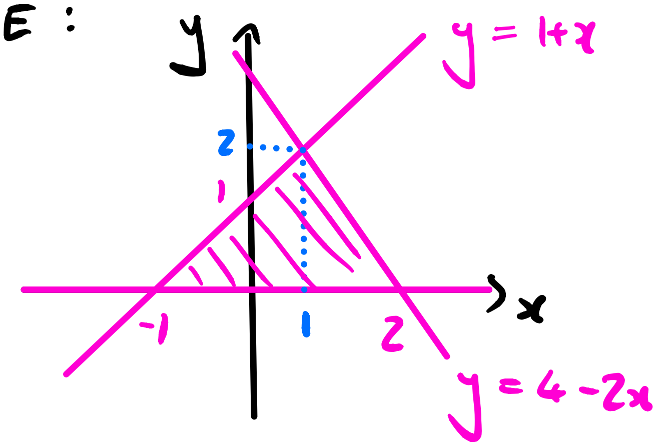
$$= 2\pi \ln \frac{2R}{R}$$

$$= 2\pi \ln 2,$$

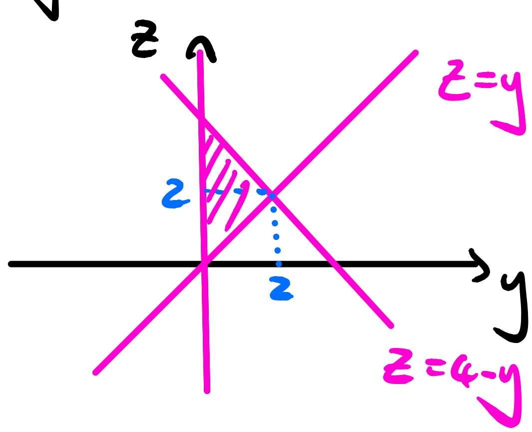
WHICH DOES NOT DEPEND ON R.

4. (16 points) Find the volume of the region bounded by the surfaces $y = 0$, $y = 1+x$, $y = 4-2x$, $z = y$ and $z = 4 - y$.

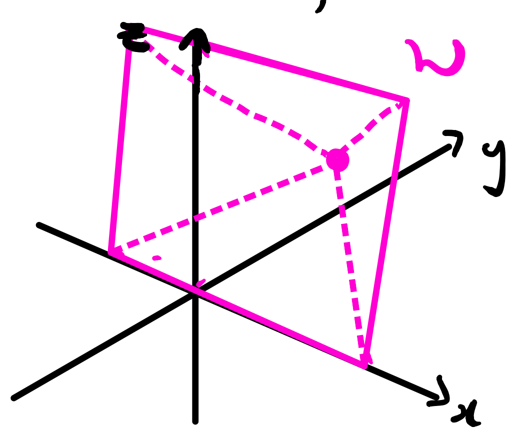
START BY SKETCHING PROJECTIONS ONTO THE (x,y) - PLANE :



AND THE (y,z) - PLANE :



IN 3D, OUR REGION, WHICH WE DENOTE BY W IS :



THEN, WE COMPUTE

$$\begin{aligned}
\text{VOLUME}(W) &= \iiint_W dV \\
&= \int_0^2 \int_{y-1}^{2-\frac{1}{2}y} \int_y^{4-y} dz\, dx\, dy \\
&= \int_0^2 \int_{y-1}^{2-\frac{1}{2}y} 4-2y \; dx\, dy \\
&= \int_0^2 (4-2y)(3-\tfrac{3}{2}y)\, dy \\
&= \int_0^2 12-12y+3y^2 \; dy \\
&= \Big[12y-6y^2+y^3\Big]_{y=0}^{y=2} \\
&= 24-24+8 \\
&= 8.
\end{aligned}$$