Quiz 3

MATH 32A-3, CALCULUS OF SEVERAL VARIABLES, FALL 2016

9

SECTION: 3A 3B 3C 3D 3E 3F (CIRCLE ONE)

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You have 10 minutes to solve the following problems. Show all of your work. To receive full credit, your answer must be neatly written and logically organized.

In what follows, consider the circle C_R of radius $R > 0$ parameterized by $\mathbf{r}(\theta) = R\langle \cos\theta, \sin\theta\rangle$ for $-\infty < \theta < \infty$.

Problem 1.(5 points.) Find the unit tangent vector $\mathbf{T}(\theta)$ and unit normal vector $\mathbf{N}(\theta)$ to $\mathbf{r}(\theta)$. Does $\mathbf{N}(\theta)$ point inside or outside of the circle?

$r(\theta) = R\langle\cos\theta, \sin\theta\rangle$

$r'(\theta) = \langle -R\sin\theta, R\cos\theta\rangle$

$\|r'(\theta)\| = \sqrt{R^2\sin^2\theta + R^2\cos^2\theta}$

$= R$

$\therefore T(\theta) = \dfrac{r'(\theta)}{\|r'(\theta)\|} = \langle -\sin\theta, \cos\theta\rangle$ //

$T'(\theta) = \langle -\cos\theta, -\sin\theta\rangle$

$\|T'(\theta)\| = 1$

$N(\theta) = \dfrac{T'(\theta)}{\|T'(\theta)\|} = \langle -\cos\theta, -\sin\theta\rangle$ //.

$N(\theta) \propto -r(\theta)$

$\therefore$ it points opposite to $r(\theta)$

$\therefore$ it points inside the circle, to the center.

Problem 2.(5 points.) Calculate the curvature $\kappa(\theta)$ of the circle C_R at $\mathbf{r}(\theta)$.

$\kappa(\theta) = \dfrac{\|r''(\theta) \times r'(\theta)\|}{\|r'(\theta)\|^3}$

$= \dfrac{\|\frac{d}{dx}\langle -R\sin\theta, R\cos\theta\rangle \times \langle R\sin\theta, R\cos\theta\rangle\|}{\|\langle -R\sin\theta, R\cos\theta\rangle\|^3}$

$= \dfrac{\|\langle -R\cos\theta, -R\sin\theta\rangle \times \langle -R\sin\theta, R\cos\theta\rangle\|}{R^3}$

$= \dfrac{-R^2\cos^2\theta - R^2\sin^2\theta}{R^3} = -\dfrac{R^2(\cos^2\theta + \sin^2\theta)}{R^3}$

$= + \dfrac{1}{R}$ //

-1

absolute value