MATH 32A Fall 2017
Midterm 1

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10/16/2017

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This exam contains 7 pages (including this page) and 5 questions. Total of points is 120. Work neatly and show all your work, including all intermediate steps and also explain in words how you are solving a problem. Partial credits are available for most problems. Calculators are not allowed in this exam. You have 50 minutes.

Grade Table (for instructor use only)

Question	Points	Score
1	20	20
2	20	13
3	20	10
4	40	21
5	20	10
Total:	120	74

1. (20 points) (a) (10 points) Are the vectors $(1,4,7)$, $(2,5,8)$, $(3,6,9)$ in the same plane? Show the reason;

   (b) (10 points) If yes, find the equation of such a plane.

36 − 42

21 − 9

6 − 12

32 − 35

14 − 8

a) $\vec{u} = \langle 1,4,7 \rangle$

$\vec{v} = \langle 2,5,8 \rangle$

$\vec{w} = \langle 3,6,9 \rangle$

If $\vec{u}$, $\vec{v}$, and $\vec{w}$ are on the same plane, they should all have cross products that face the same direction. ✓

$\vec{u}\times\vec{v} = \vec{v}\times\vec{w} = 2(\vec{u}\times\vec{w})$

$\vec{u}\times\vec{v} = \langle -3, 6, -3 \rangle$

$\vec{u}\times\vec{w} = \langle -6, 12, -6 \rangle$

$\vec{v}\times\vec{w} = \langle -3, 6, -3 \rangle$

∴ they all have parallel normal vectors so they all lie in the same plane.

b) Point $(1,4,7)$

$\vec{n} = \langle -3, 6, -3 \rangle$

P_1: $0 = \langle -3,6,-3 \rangle \cdot (\langle x,y,z \rangle - (1,4,7))$

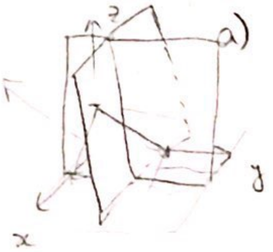
$0 = -3x + 6y - 3z - (-3 + 24 - 21)$

$0 = -3x + 6y - 3z$ ✓

$\langle 0,1,1 \rangle$ = nor x = 0

2. (20 points) (a) (10 points) Let $\mathcal{L}$ denote the intersection of the planes $x = 1$ and $y+z = 2$. Find parametric equations for the line $\mathcal{L}$;

   (b) (10 points) Find parametric equations for the line through $P_0 = (3, -1, 1)$ perpendicular to the plane $3x + 5y - 7z = 29$.



a) $P_1: x_1 = 1$

$P_2: y+z=2$

$\vec{n}_1 = \langle 1, 0, 0 \rangle$

$\vec{n}_2 = \langle 0, 1, 1 \rangle$

$\vec{n}_1 \times \vec{n}_2 = \langle 0, -1, 1 \rangle$

$\mathcal{L}$ = Point + $t\langle 0,-1,1 \rangle$

$\mathcal{L} = \langle 1$

~~The planes do not intersect~~

−7, the planes do intersect.

$x = 1$

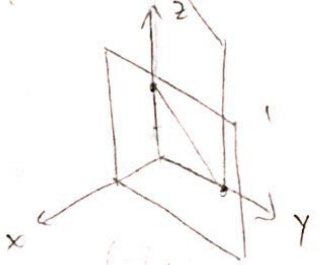
$y+z=2$

$y = 2 - z$

let $y = 0$

$z = 2$

let $z = 0$



$\mathcal{L} = (1,1,1) + t\langle 0,-1,1 \rangle$

b) Point: $(3,-1,1)$

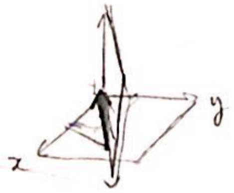
normal vector to $3x+5y-7z=29$ $= \langle 3,5,-7 \rangle$

$\boxed{\ell(t) = (3,-1,1) + t\langle 3,5,-7 \rangle}$ ✓

3. (20 points) Find all planes in $\mathbb{R}^3$ whose intersection with the xy-plane is the line $r(t) = t(2, 1, 0)$.

xy plane : $z = 0$

~~zx~~



$a \quad b \quad 0$

$p \quad q \quad 0$

$P + t(\vec{n}_1 \times \vec{u}_2) = t(2,1,0)$

$\vec{n}_1 = \langle 0,0,1 \rangle$

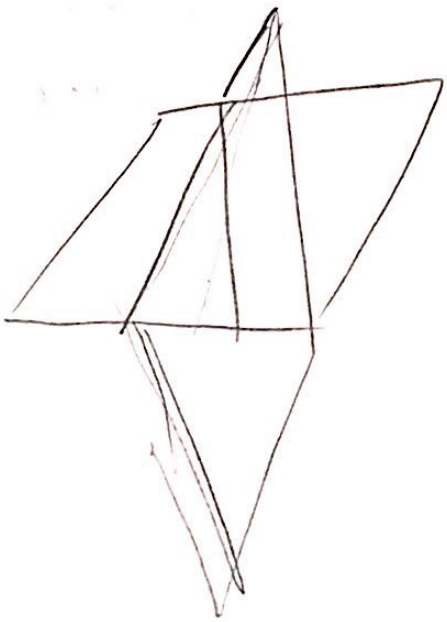
$0, \; 0, \; 1$

$a \quad b \quad c$

$\vec{n}_1 \times \vec{n}_2 = (2,1,0)$

$\langle 0,0,1 \rangle \times \langle a,b,c \rangle = \langle 2,1,0 \rangle$

$\langle -b, a, 0 \rangle = \langle 2,1,0 \rangle$



$2 = -b \qquad b = -2$

$a = 1 \qquad c = 0$

$n_2 = \langle 1,-2,0 \rangle$

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$0 = \langle 1,-2,0 \rangle \cdot \langle x,y,z \rangle$

$0 = x - 2y$

$0 = ax - 2by + cz$

normal $= \langle 2,1,0 \rangle \times \langle 0,0,1 \rangle$

$=$

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4. (20 points) (a) (15 points) Solve the equation $(1,1,1) \times \vec{v} = (1,-1,0)$ where $\vec{v} = (x,y,z)$;

(b) (5 points) Explain geometrically why $(1,1,1) \times \vec{v} = (1,0,0)$ has no solution where $\vec{v} = (x,y,z)$.

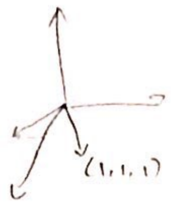
a) $\langle 1,1,1\rangle \times \langle x,y,z\rangle = \langle 1,-1,0\rangle$

$(1,1,1)\cdot(1,-1,0) = 1-1+0 = 0$

i j k
1 1 1
x y z

$z=2 \quad y=1$

$z-y=1$
$x-z=-1$
$y-z=-1$



$\langle z-y, x-z, y-x\rangle = \langle 1,-1,0\rangle$

① $z-y=1 \Rightarrow z=1+y$

② $x-z=-1 \Rightarrow z=1+x$

③ $y-x=0 \Rightarrow y=x$

$z=1+y$
$z=1+y$
$y=x$

$y=z$

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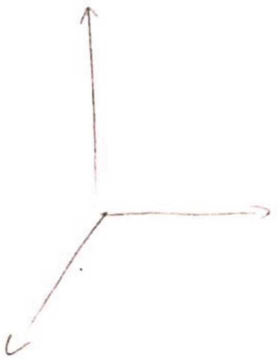
infinite amount of solutions

① $z=1+x$

① $z=1+x$

b)



if $(1,1,1)\times\vec{v} = (1,0,0)$, $(1,1,1)\perp(1,0,0)$

$\langle 1,1,1\rangle\cdot\langle 1,0,0\rangle = 1 \neq 0$

$\therefore \langle 1,1,1\rangle$ is not orthogonal to $\langle 1,0,0\rangle$,

$\therefore$ no solution exists

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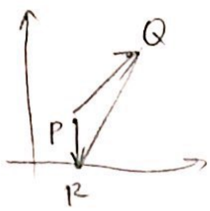
5. (20 points) Use the cross product to find the area of the triangle with vertices $P = (1,1)$, Q=(2,2) and $R = (1,0)$.



0 + 1

$\vec{PR} = (0,-1)$
$\vec{PQ} = (1,1)$

Show more work

Area $= \frac{\|PR \times PQ\|}{2}$

$= \boxed{\frac{1}{2}}$

−10